Dr.Babasaheb Ambedkar Open University

DOR

DIPLOMA IN OPERATION RESEARCH

Block

1

Basics of Operation Research

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Unit : 1 : Introduction to operation research

Introduction:

Operation Research is not a new concept infact, operation research activities started during the second world war to solve the complex and difficult military operations but then after with the knowledge of its multiple benefits, it was widely used in the civilians life to solve all kinds of problems i.e. business as well as non business. Business operations becomes very difficult & complex and thus the role of operation research activities has become very important to find the solutions in different critical problems by finding all possible alternative and choosing best among it. During this first chapter, few of the basic aspects like the need, occurrence and usefulness of operation research has been discussed.

Structure of the Chapter:

1.1 Objectives:

By the end of this chapter the students will learn about

- Definition of O.R.
- History of O.R.
- Scientific methods of O.R.
- Advantages and Shortcomings of O.R.
- Features and Scope of O.R.
- Different O.R. models

1.2 Operations Research : Definition

Precise or limited definition of Operation Research is not possible because of the wide scope of application of operations research. The definition of OR are given below:

Operations research is concerned with scientifically deciding how to best design and operate man-machine systems usually requiring the allocation of scarce resources. Operations Research Society, America

"O.R Is a scientific approach to problems solving for executive management." —HM. Wagner

(i) "O.R. Is the art of giving bad answers to problems which otherwise have worse answers."

1.3 Operations Research and Decision-Making- together

Decision making is crucial aspect of Managerial role. It comes at various stage of the business. For such decision making they have to consider various factors and effect of changes in each of such factors. And hence it becomes a huge task to consider the effects of various factors which affect decision making for a single individual. Business matters have become uncertain. The business man has to choose the best option available out of variety of options / opportunities available. He has to make decision regarding best possible investments of its efforts. timing and money. For the purpose he has to consider various factors affecting the decision. A technology change is a day to day affair now. Each day new inventions make the current technologies obsolete. Hence, in order to effectively address these problems, decision-makers may not afford to make decisions by simply applying their personal experiences, guesswork or intuition, because the results of wrong decisions are serious and costly. For example, entering the wrong markets, producing the wrong products. offering to inappropriate segment of people, providing inappropriate services, etc.. may have major adverse results for organizations. Considering all these factors the use of operation research techniques and computers has widened in decision-making process. Decision maker has the method of evaluating every possible alternative and the all possible results with the help of Operations research & its various techniques to know. But we can not say that the decision making is simple about the application of operation research techniques. So, in simple words Operation Research is concerned with selecting the best alternative out of many alternatives available to it after considering all the alternatives. Moreover, it is also concerned with the best distribution of scarce resources.

1.4 The History of Operations Research-in brief

Now we will have an idea of the development of Operation Research. It will be at the discussed at world level as well as at India level. It has been accepted that operations research came into existence during Second World War when managing the critical resources has become very important. The term 'operations research' was used as a result of research on military operations during the Second World War. Since the war involved strategic and tactical problems which were highly complicated,- it become difficult to get the expected result from the single individuals. And so to resolve the problem, groups of individuals who were masters in various areas like in mathematics, economics, accountants, statistics and probability theory, engineering, behavioral, and physical science were formed as special units and considered as specialist within the armed forces to deal with various strategic and tactical problems of various military operations. And with the best possible results by the Operation Research during second world war it was started with the wide use in the civilian life as well as, it was started widely in business operation. O.R is addressed to managerial decision-making or problem solving. A major premise of O.R is that decision-making, irrespective of the situation involved, can be considered as a general systematic process.

In simple words, in the beginning stage, O.R. was used in the second world war but gradually, it became useful tool for decision making and problem solving in civilian life.

1.5 Nature and Significance of Operations Research with Steps

O.R employs scientific methods for the purpose of solving problems. It is a formalized process of reasoning. The operations research approach is mainly useful in balancing conflicting goals where there are more than one alternative courses of action available and the decision-makers is required to select the best among all the possible results. In a real sense, the optimum decision should be one that can produce best for the organization as a whole. Operation Research considers all available factors, conditions and possibility. So it tries to resolve the conflicts of interest among various sections of the organization and seeks the best and optimal solution, which may be acceptable to all the departments but and as well as it is in the interest of the organization as a whole. Moreover, OR is concerned with providing the decision-maker with decision tools, alternatives and options so the decision maker can taken an informed decision. O.R attempts to locate the best or optimal solution to the problem under consideration. For this purpose, it is necessary that a measure of effectiveness is defined which is based on the goals of the organization. This measure is then us as the basis to. compare the alternative courses of action.

Following steps are necessary for the successful application of OR techniques to solve a problem.

1. Constructing various mathematical, economic or statistical model of the problem under consideration.

2. Observing the relationships among different variables within the problem itself and on the basis of those possible results of such problems.

3. Suggesting objective function in order to evaluate possible courses of action, acts or strategies.

1.6 Procedure

The most important feature of operations research is building of decision models. There are three phases in OR procedure is based.

Judgment Phase

This phase includes:

(i) identification of the problem

(ii) Selection of objectives and various variables related to the problem

(iii) deciding the intensity of desirability

(iv) Formulation of a model of the problem, abstracting the essential information, so that a solution to the decision-maker's goals may be obtained

Research Phase

This phase utilizes:

(i) data collection for understanding of the problem

(ii) formulation of model

(iii) observations to test the model on the basis of additional available data

(iv) Analysis of the available information and verification of the model using pre-established measures of desirability

(v) Estimation of various results from the model.

Action Phase

This is the last phase in the problem which is demanding action. This phase contains making suggestion for implementing the decision by an individual who is in fact in a position to implement results. That is the reason we are referring the phase as an Action Phase.

1.7 Opportunities and Shortcomings of Operations Research Approach

A few opportunities and shortcomings of the OR approach are listed below.

Opportunities:

- It forces the decision-maker to be quite explicit about his objective and assumptions.
- It makes the decision-maker to observe very carefully just what variables influence the decisions.

Shortcomings

- Often solution to a problem is derived by making assumptions and thus, such solutions have limitations of those assumptions inherited in it.
- It is very difficult to represent the realistic and proper situations in the models in which decisions must be made.
- Sometimes decision-maker may not be fully aware of the limitations of the models that he is using.

1.8 Features of Operations Research Solution

- Solution to the problem should be within the budget as well as the solution should not cost more than the budgeted expenditure. Moreover, all the solutions should be useful for a certain reasonable period of time under the conditions for which it was designed.

1.9 Scope of Operations Research

The problems. Industrial. Government or Business which are analyzed and resolved by OR approach have been arranged by its function as follows.

Finance and Accounting

- Dividend policies, investment, Break-even analysis, capital budgeting and portfolio management, auditing, balance sheet and cash flow analysis, cost allocation and control and financial planning

Marketing

- Advertising, media planning, Selection of product-mix, Sales effort allocation, staff productivity, marketing and export planning, Best time to launch a new product

Purchasing, Procurement and Exploration

- Transportation planning, Replacement policies, best buy situation & Optimal buying with price quantity discount

Production Management Facilities Planning

- Planning and scheduling, layout and engineering design, Location and size of warehouse or new plant, Transportation.

Manufacturing

- Employment, training, Production Planning, Allocating R&D budgets, Assembly line, blending, purchasing, inventory control and reducing wastage – scrap.

Personnel Management

- Manpower planning, Scheduling of training program to maximize skill development, wage/salary administration

Government

- Economic planning, Urban and housing problems, natural resources, social planning and energy, police, pollution control, etc.

1.10 Basic Operations Research Models

Several OR models or techniques may be grouped into some basic categories as given below.

Allocation Models

Allocation models are used to allocate all available resources to activities in accordance to its importance in such a way that best results are produced and objectives are optimized. e.g. linear programming problem models, assignment problem models, transportation problems.

Inventory Models

Inventory models deal with the problem of determination of how much to order & at what point of time to place an order. The main objective is to minimize the sum of three conflicting inventory costs: The cost of holding or carrying extra inventory, the cost of shortage or delay in the delivery of items when it is needed, a cost of ordering or set-up.

Queuing Models

These models have been used to establish coordination between costs of providing service and the waiting time of a customer in the queuing system such as arrival process, queue structure and service process, average length of waiting time, average time spent by the customer in the line, traffic intensity, etc., of the waiting system. This is very useful in customer service oriented industry.

Game Theory Models

These models have been used to see the behavior of two or more players who compete for the achievement of conflicting goals. These models are based on several factors such as number of competitors, sum of loss or gain and the type of strategy which would yield the best or the worst outcomes.

Network Models

These models are used by the management in planning, controlling and scheduling of large-scale projects. PERT / CPM uses the techniques of the identification of the critical path. This model reduces the lag time between activities. This model is emphasizing on the early completion of the project.

Sequencing Models

The sequencing models helps in deciding sequences in the problem and such problem arises whenever there is a problem in determining order in which a number of tasks may be performed.

Replacement Models

These models are used when one must decide the best time to replace equipment.

Decision Analysis Models

These models deal with the determination of an optimal course of action given the possible payoffs and their associated probabilities of occurrence.

1.11 Exercise

1. Define Operation Research and state its relations with decision making?
2. Describe the history of Operation Research in brief?
3. What are the opportunities and shortcomings of Operation Research?
4. State the uses of Operation Research in various ares?
5. Describe various Operation Research models?

Unit: 2: Linear Programming:

Introduction

Decision making is required at each and every stage of the business. Such decisions are required to be taken under many constrain situations. So, decision makers have to take their decision under many certain limitations. To make the best decision under the limitation they need to solve the problem mathematically and for the purpose they use linear programming models. In a decision-making environment, model formulation is important because it shows the essence of business decision problem. The term formulation is used to mean the process of building the verbal description and numerical data into mathematical expressions, which represents the relationship among decision factors, objectives and restrictions on the use of available resources. Linear Programming (LP) is a mathematical modeling technique useful for economic allocation of 'scarce' or 'limited' resources, such as labour , material, machine, time, warehouse space, capital, energy, etc., to several competing activities, such as products, services, jobs, new equipment, projects, etc., on the basis of a given criterion of optimality. So, linear programming is used to give the best results under the worse conditions. It is being used extensively in all functional areas of management, agriculture, military operations, oil refining, pollution control, research and development, health care systems, etc.

Structure of the Chapter:

2.1 Objectives:

By the end of this chapter the students will learn about

- Structure of Linear programming model
- Application areas of linear programming
- Guidelines in linear programming model formulation
- Graphical solution of linear programming models

2.2 Structure of Linear Programming Model

General Structure of LP Model

The procedure of linear programming is starting with the determination of the problem then determination of desired result (Objective Function), available alternatives and proper solution. The general structure of LP model consists of three basic elements or components.

Decision variables (activities): Decision maker need to evaluate all the available alternatives (courses of action) for arriving at the optimal value of objective function. For this, we pursue certain activities (also called decision variables) usually denoted by $x_1, x_2, \ldots, x_n$. Changes in such variable may change the outcome. Moreover, in an LP model all decision variables are continuous, controllable and non-negative. That is, $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$.

The objective function: The objective function of each LP problem is expressed in terms of decision variables to optimize the criterion of optimality such as profit, cost, revenue, sales, distance etc. In its general form, it is represented as:

Optimize (Maximize or Minimize) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

where Z is the measure-of-performance variable (needs to be optimize), which is a function of $x_1,x_2, \ldots, x_n$. Quantities $c_1,c_2, \ldots, c_n$ are parameters that represent the contribution of a unit of the respective variable $x_1,x_2, \ldots, x_n$ to the measure-of-performance Z.

The constraints There are always certain limitations on the use of resources, e.g. labour, machine, raw material, space, money, other expenses etc., that limit the degree to which an objective may be achieved. Such constraints must be expressed as decision variables. The solution of an LP model must satisfy these constraints.

2.3 Assumptions of Linear Programming

The following four basic assumptions are necessary for all linear programming models.

In all LP models, it is assumed, that all parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable, demand of the unit in market and consumption of resources by a unit of decision variable must be known and may be constant.

The solution values of decision variables are allowed to presume continuous values. Hence, if any of the variables may assume only integer values or are limited to discrete number of values, LP model is no longer applicable.

The value of the objective function and the total amount of each resource used must be equal to the total sum of the respective individual contributions (profit or cost) by decision variables.

The amount of each resource used and its contribution to the profit (or cost) in objective function must be proportional to the value of each decision variable.

2.4 Advantages of Linear Programming

Following .are certain advantages of linear programming:

- Linear programming helps in getting the optimum and best use of productive resources.
- It indicates how a decision-maker should employ his productive resources in best possible manner by selecting and allocating these resources to proper function.
- Linear programming techniques improve the quality of decisions. The decision-making approach of the user of this technique becomes more scientific and objective.
- Linear programming techniques provide possible and practical solutions under given circumstances for optimum decision.
- Linear programming also helps in reforming of a basic plan for changing conditions.

2.5 Limitations of Linear Programming,

In spite of having many advantages and wide areas of applications, there are some limitations associated with this technique. Basically, the assumptions used in linear programming may be treated as the limitations of the programming. These are given below:

- Linear programming treats all relationships among decision variables as linear. However, in fact, neither the objective functions nor the constraints in real-life situations concerning business, industrial or any other problems are linearly related to the variables.
- While solving an LP model, there is no guarantee that we will get integer valued solutions. It required to be rounded off the solution to the nearest integer and that will not yield an optimal solution.
- Parameters appearing in the model are assumed to be constant but in real-life business or industrial situations, they are frequently neither known nor constant.
- It deals with only single objective, whereas in practical business or industrial situations we may come across conflicting multi-objective problems.

2.6 Application Areas of Linear Programming

Linear programming is the most widely used technique of decision-making in business and industry and in various other fields. Here, we will discuss a few of the broad application areas of linear programming.

Agricultural Applications

Linear programming may be applied in agricultural planning, e.g. allocation of limited resources such as acreage, labour, water supply and working capital, usage of land for particular product etc., in a way so as to maximize net revenue.

Production Management

It helps to determine product mix. A company may produce several different products, each of which requires the use of limited production resources. In such cases, it is essential to determine the quantity of each product to be produced knowing its marginal contribution and amount of available resource used by it. So it's very crucial for deciding the correct product mix.

Financial Management

It helps in portfolio selection. This deals with the selection of specific investment activity among several other activities. The objective is to find the allocation which maximizes the total expected return or minimizes risk under certain limitations. It is also helpful in profit planning. So this is helpful in minimizing the risk and maximizing the profit.

Marketing Management

It helps in physical distribution. Linear programming determines the most economic and efficient manner of locating manufacturing plants and distribution centers for physical distribution. It is also used in media selection. Thus proper distribution of the product to its most favorable market makes possible by Linear Programming.

Personnel Management

It helps in staffing problem Linear programming is used to allocate optimum manpower to a particular job so as to minimize the total overtime cost or total manpower. Allocation of manpower to maximize its productivity has be done by Linear Programming.

2.7 General Mathematical Model of Linear Programming Problem

The general linear programming problem (or model) with n decision variables and m constraints may be stated in the following form.

Find the values of decision variables $x_1, x_2, \ldots, x_n$ so as to

Optimize (Max. or Min.) $Z = c_1,x_1 + c_2x_2 + \ldots + c_nx_n$

subject to the linear constraints,

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\ (\leq, =, \geq)\ b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\ (\leq, =, \geq)\ b_2 \\
\vdots \qquad \vdots \qquad \quad \vdots \qquad & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\ (\leq, =, \geq)\ b_m \\
x_1, x_2, \ldots, x_n &\geq 0
\end{aligned}$$

and

The above formulation may also be expressed in a compact form using summation sign.

$$\text{Optimize (Max. or Min.) } Z = \sum_{j=1}^{n} c_j x_j \qquad \text{(objective function)} \qquad (1)$$

subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij} x_j \ (\leq, =, \geq)\, b_i; \ i = 1, 2, \ldots, m \qquad \text{(constraints)} \qquad (2)$$

and

$$x_j \geq 0; \ j = 1, 2, \ldots, n \qquad \text{(non-negativity conditions)} \qquad (3)$$

Here, the c_j's are coefficients representing the per unit contribution of decision variable x_j to the value of objective function. The a_{ij}'s are called the technological coefficients or input-output coefficients. These represent the amount of resource, say ; consumed per unit of variable (activity) x_j. In the given constraints, the a_{ij}'s may be positive, negative or zero. The b_i represents the total availability of the ith resource. The term resource is used in a very general sense to include any numerical value associated with the right hand side of a constraint. It is assumed that $b_i > 0$ for all i. However, if any $bi < 0$, then both sides of constraint i may be multiplied by -1 to make $b_i > 0$ and reverse the inequality of the constraint. In, the general LP problem, the expression ($\leq$, =, $\geq$) means that in any specific problem each constraint may take only one of the three possible forms:

(i) less than or equal to the value ($\leq$)

(ii) greater than or equal to the value ($\geq$)

(iii) equal to the value (=)

2.7 Guidelines on Linear Programming Model Formulation

Steps of linear programming model formulation are summarized as follows:

Step 1

Express each constraint in words. For this first see whether the constraint is of the form $\geq$ (at least as large as), or of the form (no larger than) or = (exactly equal to). Then express the objective function verbally. Steps (a) and (b) should then allow you to verbally identify the decision variables.

Step 2

For solving a problem, we need to identify the problem data so as to provide the actual values for the decision variables and put it in the Linear Programming Model Function.

Step 3

Express the constraints verbally in terms of requirements and availability of each resource. Convert the verbal expression of the constraints imposed by the resource availability as linear equality or inequality in terms of the decision variables defined in Step 1.

Step 4

Identify whether the objective function is to be optimized (i.e .maximized or minimized). Then express it verbally, such as, maximize total profit/cost and then convert it into a linear mathematical expression in terms of decision variables multiplied by their profit or cost contributions.

2.8 Linear Programming: The Graphical Method

Important Definitions

Solution: The result of the function, the set of values of decision variables x_j (j=1,2,..., n) which satisfy the constraints of an LP problem is said to constitute solution to that LP problem.

Feasible solution: The set of values of decision variables x_j (j= 1,2, . .., n) which satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the feasible solution to that LP problem.

Infeasible solution: Being reverse of the feasible solution, this set of values of decision variables x_j (j = 1, 2, . . ., n) which do not satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the infeasible solution to that LP problem.

Basic solution: For a set of m simultaneous equations in n variables (n > m), a solution obtained by setting (n - m) variables equal to zero and solving for remaining m equations in m variables is called a basic solution.

The (n - m) variables whose value did not appear in this solution are called non-basic variables and the remaining m variables are called basic variables.

Basic feasible solution A feasible solution to an LP problem which is also the basic solution is called the basic feasible solution. That is, all basic variables assume non-negative values.

2.9 Extreme Point Enumeration Approach

The steps of the method are summarized below:

Step 1 Develop LP model

State the given problem in the mathematical LP model as shown earlier.

Step 2 Plot constraints on graph paper and decide the feasible region

Replace the inequality sign in each constraint by an equality sign. Allocate the points on the graph paper according to the equations. Draw these straight lines on graph paper and decide each time the area of feasible solutions according to the inequality sign of the constraint. Shade the common portion of the graph that satisfies all the constraints simultaneously drawn so far. Thus we arrive at the final shaded area, called the feasible region (or solution space) of the given LP problem. Any point inside this region is called feasible solution and provides values of x_1 and x_2 that satisfy all constraints.

Step 3 Examine extreme points (corners) of the feasible solution space

(i) Determine the coordinates of each corner (extreme) point of the area of feasible region (or space) where all constraints overlap.

(ii) Calculate the value of the objective of each function at each extreme point.

(iii) Determine the extreme point of the feasible region that has optimum (the best) objective function value.

2.10 Special Cases in Linear Programming

Alternative (or Multiple) Optimal Solutions

When a given LP problem may have more than one solution yielding the same optimal objective function value. All the optimal solutions is termed as alternative optimal solution and such problem is denoted as alternative optimal solution.

Unbounded Solution

Sometimes an LP problem will not have a finite solution. This means when one or more decision variable values and the value of the objective function (maximization case) are permitted to increase infinitely without violating the feasibility condition, then the solution is said to be unbounded.

Infeasible Solution

Infeasibility is a condition that arises when no value of the variables satisfy all of the constraint simultaneously. This means, there is no unique (single) feasible region.

Redundancy

A redundant constraint is one that does not affect the feasible solution region (or space) and thus redundancy of any constraint does not cause any difficulty in solving an LP problem graphically.

2.11 Exercise

1. Describe the structure of the linear programming model?
2. State the advantages and limitations of linear programming models
3. State the application areas of linear programming models
4. State the guidelines in linear programming model formulation?
5. Write a short note on the graphical solution of linear programming problems?

2.12 Practical:

1. Akash Ltd.is engaged in producing three types of products: A, B and C. The production department produces, each day, components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The following additional information is available.

Type of Product	Profit Contribution per Unit of Product (Rs)	Assembly Time per Product (hrs)
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of product A and a total of 15 units of products B and C. Formulate this problem as an LP model so as to maximize the total profit.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	Product Type			Total
	A	B	C	
Production capacity (units)	50	25	30	
Man-hours per unit	0.8	1.7	2.5	100
Order commitment (units)	20	15 (both for B and C)		
Profit contribution (Rs/unit)	12	20	45	

Decision variables Let x_1, x_2 and x_3 = number of units of product A, B and C to be produced respectively.

The LP model

Maximize (total profit) $Z = 12x_1 + 20x_2 + 45x_3$ subject to the constraints

(i) Labour and materials constraints

$0.8x_1 + 1.7x_2 + 2.5x_3 \leq 100$

$x_1 \leq 50$

$x_2 \leq 2$

$x_3 \leq 30$

(ii) Order commitment constraints

$x_1 \geq 20$

$x_2 + x_3 > 15$

and $x_1, x_2, x_3 > 0$

2. Anuj Ltd. has two plants, each of which produces and supplies two products: A and TB. The plants may each work up to 1.6 hours a day. In plant 1, it takes three hours to prepare and pack 1,000 gallons of A and one hour to prepare and pack one quintal of B. In plant 2, it takes two hours to prepare and pack 1,000 gallons of A and 1.5 hours to prepare and pack a quintal of B. In plant 1, it costs Rs 15,000 to prepare and pack 1,000 gallons of A and Rs 28,000 to prepare and pack a quintal of B, whereas these costs are Rs 18,000 and Rs 26,000, respectively in plant 2. The company is oblized to produce daily at least 10 thousand gallons of A and 8 quintals of B.

Formulate this problem as an LP model to find out as to how the company should organize its production so that the required amounts of the two products be obtained at minimum cost.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	Product A	Product B	Total Availability (hrs)
Preparation time (hrs)	Plant 1: 3 hrs/thousand gallons	1 hr/quintal	16
	Plant 2: 2 hrs/thousand gallons	1.5 hr/quintal	16
Minimum daily production	10 thousand gallons	8 quintals	
Cost of production (Rs)	Plant 1: 15,000/thousand gallons	28,000/quintals	
	Plant 2: 1 8,000/thousand gallons	26,000/quintals	

Decision variables Let

x_1, x_2 = quantity of product A (in '000 gallons) to be produced in plants 1 and 2, respectively

x_3, x_4 = quantity of product B (in quintals) to be produced in plants 1 and 2, respectively

The LP model

Minimize (total cost) $Z= 15{,}000x_1 + 18{,}000x_2 + 28{,}000x_3 + 26{,}000x_4$ subject to the constraints

(i) Preparation time constraints

$3x_1 + x_2 \leq 16$

$2x_3+1.5x_4 \leq 16$

(ii) Minimum daily production requirement constraints

$x_1 + x_2 \geq 10$

$x_3 + x_4 \geq 8$ and

$x_1, x_2, x_3, x_4 \geq 0$

3.: Anita Ltd. produces three items A, B and C. The profit on these items are 2, and 3 respectively. These items are prepared on three machines M_1, M_2 and M_3. The details regarding uses and availabilities of these machines are given in the following table :

Machine	Hours required / unit			Maximum available hours
	A	B	C	
1	1	1	1	80
2	8	5	4	500
3	3	3	7	210

Represent this as a linear programming problem :

Solution :

Suppose x_1, x_2 and x_3 units of A, B and C are produced.

$Z = 2x_1 + 5x_2 + 3x_3$ is to be maximised.

Subject to the constraints

$x_1 + x_2 + x_3 \leq 80$

$8x_1 + 5x_2 + 4x_3 \leq 500$

$3x_1 + 3x_2 + 7x_3 \leq 210$

$x_1, x_2, x_3 \geq 0$

4.: The manager of Raja Ltd. must decide on the optimal mix of two blending processes of which the inputs and outputs per production run are as follows :

	Inputs (units)		Outputs (units)	
	Crude A	Crude B	Gasoline x	Gasoline y
Process 1	5	3	5	8
Process 2	4	5	4	4

The maximum amounts available of crude A and B are 200 units and 150 units respectively. The market requirements show that atleast 100 units of gasoline x and 80 units of gasoline y must be produced. The profits per production run of process 1 and process 2 are respectively Rs. 300 and Rs. 400.

Express this as a linear programming problem.

Solution :

Suppose the production run of process 1 is x_1, and that of process 2 is x_2. The objective function will be $Z = 300x_1 + 400x_2$ The maximum availability of crude A is 200 units.

$5x_1 + 4x_2 \leq 200$

Similarly for crude B

$3x_1 + 5x_2 \leq 150$

The market requirements of gasoline x and gasoline y are atleast 100 and 80 respectively.

$5x_1 + 4x_2 \geq 100$

$8x_1 + 4x_2 \geq 80$

Moreover x_1, x_2 maynot be negative i.e. $x_1 \geq 0$, $x_2 \geq 0$

The mathematical representation of the problem may be

Maximize $Z = 300x_1 + 400x_2$

Subject to

$5x_1 + 4x_2 \leq 200$

$3x_1 + 5x_2 \leq 150$

$5x_1 + 4*_2 \geq 100$

$8x_1 + 4*_2 \geq 80$

$x_1, x_2 \geq 0$

5. Smita Ltd. is engaged in the production of two components C_1 and C_2 used "In" radio sets. Each unit of C_1 costs the company Rs 5 in wages and Rs 5 in material, while each of C_2 costs the company Rs 25 in wages and Rs 15 in material. The company sells both products on one-period credit terms, but the company's labour and material expenses must be paid in cash. The selling price of C_1 is Rs 10 per unit and of C_2 it is Rs 70 per unit. Because of the strong monopoly of the company for these components, it is assumed that the company may sell at the prevailing prices as many units as it produces. The company's production capacity is, however, limited by two considerations. First, at the beginning of period 1, the company has an initial balance of Rs 4,000 (cash plus bank credit plus collections from past credit sales). Second, the company has available in each period 2,000 hours of machine time and 1,400 hours of assembly time. The production of each C_1 requires 3 hours of machine time and 2 hours of assembly time, whereas the production of each C_2 requires 2 hours of machine time and 3 hours of assembly time. Formulate this problem as an LP model so as to maximize the total profit to the company.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	Components		
	C_1	C_2	Total Availability
Budget (Rs)	10/unit	40/unit	Rs 4,000
Machine time	3 hrs/unit	2 hrs/unit	2,000 hours
Assembly time	2 hrs/unit	3 hrs/unit	1,400 hours
Selling price	Rs 30	Rs 70	
Cost (wages + material) price	Rs 10	Rs 40	

Decision variables Let x_1 and x_2 = number of units of components C_1 and C_2 to be produced, respectively.

The LP model

Maximize (total profit) Z = Selling price - Cost price

$= (30 - 10)\,x_1 + (70 - 40)\,x_2 = 20x_1 + 30x_2$

subject to the constraints

(i) The total budget available constraint

$10x_1 + 4x_2 \leq 4{,}000$

(ii) Production time constraint

$3x_1 + 2x_2 \leq 2{,}000$

$2x_1 + 3x_2 < 1{,}400$

and $x_1, x_2 \geq 0$

6. Aniket Ltd. has two grades of inspectors 1 and 2, who are to be assigned for a quality "control inspection. It is required that at least 2,000 pieces be inspected per 8-hour day. Grade 1 inspector may check pieces at the rate of 40 per hour, with an accuracy of 97 per cent. Grade 2 inspector checks at the rate of 30 pieces per hour with an accuracy of 95 per cent.

The wage rate of a Grade 1 inspector is Rs 5 per hour while that of a Grade 2 inspector is Rs 4 per hour. An error made by an inspector costs Rs 3 to the company. There are only nine Grade 1 inspectors and eleven Grade 2 inspectors available in the company. The company wishes to assign work to the available inspectors so as to minimize the total cost of the inspection. Formulate this problem as an LP model so as to minimize daily inspection cost.

LP model formulation The data of the problem is summarized as follows:

	Inspector	
	Grade 1	Grade 2
Number of inspectors	9	11
Rate of checking	40 pieces/hr	30 pieces/hr
Inaccuracy in checking	1 - 0.97 = 0.03	1 - 0.95 = 0.05
Cost of inaccuracy in checking	Rs 3/piece	Rs 3/piece
Wage rate/hour	Rs 5	Rs 4
Duration of inspection = 8 hrs per day		
Total pieces which must be inspected = 2,000		

Decision variables Let x_1 and x_2 = number of Grade 1 and 2 inspectors to be assigned for inspection, respectively.

The LP model

Hourly cost of each of Grade 1 and 2 inspectors may be computed as follows:

Inspector Grade 1: Rs $(5 + 3 \times 40 \times 0.03)$ = Rs 8.60

Inspector Grade 2: Rs $(4 + 3 \times 30 \times 0.05)$ = Rs 8.50 Based on the given data, the linear programming problem may be formulated as follows:

Minimize (daily inspection cost) $Z = 8\,(8.60x_1 + 8.50x_2) = 68.80.x_1 + 68.00x_2$ subject to the constraints

(i) Total number of pieces that must be inspected in an 8-hour day constraint

$$8 \times 40x_1 + 8 \times 30x_2 \geq 2000$$

(ii) Number of inspectors of Grade 1 and 2 available constraint

$$x1 < 9;\ :x2 < 11 \text{ and } x1, x2 \geq 0$$

7. Avishkar Ltd. produces three types of parts for automatic washing machines. It **purchases casting of the** parts from a local foundry and then finishes the part on drilling. **shaping and polishing** machines.

The selling prices of parts A, B and C, respectively are Rs 8, Rs 10 and Rs 14. All parts made may be sold. Castings for parts A, B and C, respectively cost Rs 5, Rs 6 and Rs 10.

The shop possesses only one of each type of machine. Costs per hour to run each of the three machines are Rs 20 for drilling, Rs 30 for shaping and Rs 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table:

Machine	Capacity per Hour		
	Part A	Part B	Part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40'	30	40

The management of the shop wants to know how many parts of each type it should produce per hour in. order to maximize profit for an hour's run. Formulate this problem as an LP model so as to maximize-total profit to the company.

LP model formulation Let x_1, x_2 and x_3 = numbers of type A, B and C parts to be produced per hour, respectively.

Profit must allow not only for the cost of the casting but also for the cost of drilling, shaping, and polishing. Since 25 type A parts per hour may be run on the drilling machine at a cost of Rs 20, then Rs 20/25 = Re 0.80 is the drilling cost per type A part. Similar reasoning for shaping and polishing gives

Profit per type A part = $(8 - 5) - ((20/25) + 30/25) + (30/40)) = 0.25$

Profit per type B part = (10 - 6) – ((20/40)+(30/20)+(30/30)) = 1

Profit per type C part = (14- 10) – ((20/25)+(30/20)+(30/40))=.95

On the drilling machine, one type A part consumes 1/25th of the available hour, a type B part consumes 1/40th, and a type C part consumes 1/25th of an hour. Thus, the drilling machine constraint is

$$(x1/25) + (x2/40) + (x3/25) \leq 1$$

Similarly, other constraints may be established.

The LP model

Maximize (total profit) $Z = 0.25\, x1 + 1.00\, x_2 + 0.95\, x_3$

subject to the constraints

(i) Drilling machine constraint

$$(x1/25) + (x2/40) + (x3/25) \leq 1$$

(ii) Shaping machine constraint

$$(x1/25) + (x2/20) + (x3/20) \leq 1$$

(iii) Polishing machine constraint

$$(x1/40) + (x2/30) + (x3/40) \leq 1$$

$$x1, x2, x3 \geq 0$$

8. Nishit Ltd. produces two pharmaceutical products: A and B. Production of both-products requires the same process, I and II. The production of B results also in a by-product C at no extra cost. The product A may be sold at a profit of Rs 3 per unit and B at a profit of Rs 8 per unit. Some of .this by-product may be sold at a unit profit of Rs 2, the remainder has to be destroyed and the destruction cost is Re 1 per unit. Forecasts show that only up to 5 units of C may be sold. The company gets 3 units of C for each unit of B produced. The manufacturing times are 3 hours per unit for A on pro'cess I and II, respectively, and 4 hours and 5 hours per unit for B on process I and II, respectively.. Because the product C results from producing B, no time is used in producing C. The available time's are 18 and 21 hours of process I and II, respectively. Formulate this problem as an LP model to determine the quantity of A and B which should be produced, keeping C in mind, to make the highest total profit to the company.

LP model formulation The data of the problem is summarized as follows:

Constraints/Resources	Time (hrs) Required by			Availability
	A	B	C	
Process I	3	4	—	18 hrs
Process II	3	5	—	21 hrs
By-product ratio from B	—	1	3	5 units (max. units that may be sold)
Profit per unit (Rs)	3	8	2	

Decision variables Let

$X1, x2$ its of product A and B to be produced, respectively

x_3, x_4 = units of product C to be produced and destroyed, respectively.

The LP model

Maximize (total profit) $Z = 3x1 + 8x_2 + 2x_3 - x_4$ subject to the constraints

(i) Manufacturing constraints for products A and B

$3x1 + 4x2 \leq 18$

$3x1 + 5x_2 < 21$

(ii) Manufacturing constraints for by-product C

$x_3 < 5$

$-3x_2 + x3 + x_4 = 0$

$x1, x_2, x_3, x_4 > 0$

9 : Solve the following L.P. Problem for Mihir Ltd. Find x_1, x_2 such that

$5x_1 + 10x_2 \leq 50$

$x_1 + x_2 \geq 1$

$x_2 \leq 4, x_1, x_2 \geq 0$

and $Z = x_1 + x_2$ is minimum.

Ans. : Changing inequalities into equations, we get

$5x_1 + 10x_2 = 50$

$x_1 + x_2 = 1$

$x_2 = 4$

$5x_1 + 10x_2 = 50$ $x_1 + x2 = 1$ $x_2 = 4$

x_1	0	1
x_2	1	0

x_1	0	10
x_2	5	0

The convex polygon ABCDE is obtained from the given inequalities. The vertices of the convex polygon ABCDE give the feasible solutions. We shall find the values of Z at each of the vertices.

At A (0, 1) $Z = 0 + 1 = 1$*

At B (0, 4) $Z = 0 + 4 = 4$

At C (2, 4) $Z = 2 + 4 = 6$

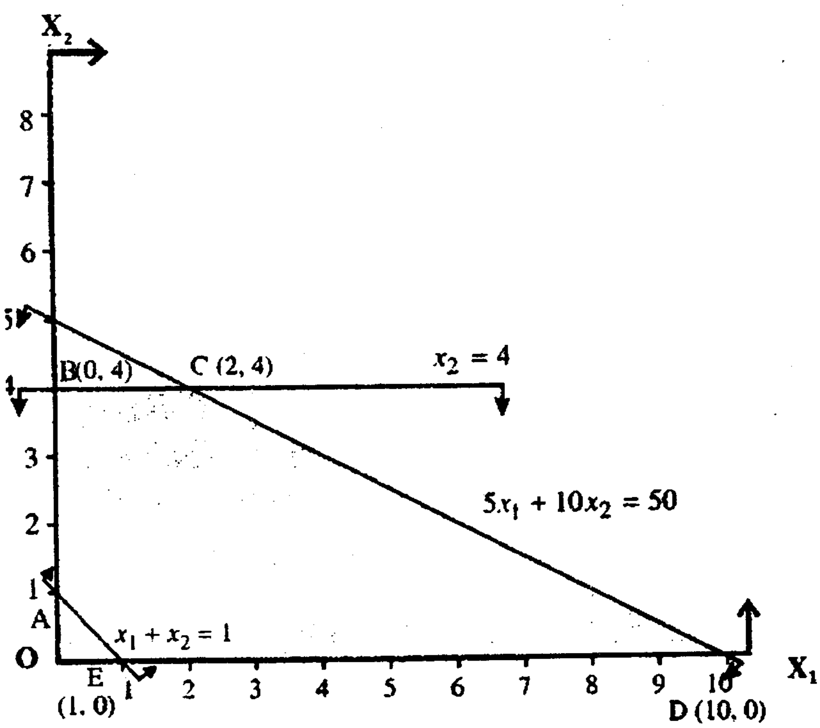
At D (10, 0) $Z = 10 + 0 = 10$

At E (1,0) $Z = 1+0=1$*

Thus the value of Z is minimum at A and E, hence two solutions of the given L.P. problem are as follows :

(i) $x_1 = 0, x_2 = 1. Z_{min} = 1$

(ii) $x_1 = 1, x_2 = 0, Z_{min} = 1$



10: Find the values of x_1, x_2, such that $Z = 3x_1 + 4x_2$ is maximum subject to the following constraints for Tejas Ltd.:

$$2x_1 + 5x_2 \leq 120$$

$$4x_1 + 2x_2 \leq 80$$

$$x_1, x_2 \geq 0$$

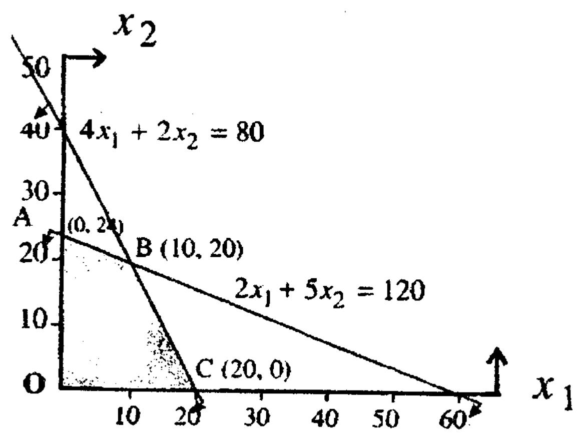
Ans.: The following equations may be obtained from the given inequalities :

$2x_1 + 5x_2 = 120$

x_1	0	60
x_2	24	0

$4x_1 + 2x_2 = 80$

x_1	0	20
x_2	40	0



The convex region OABC is obtained from the given inequalities. The vertices of this region give the feasible solutions. We shall find the values of Z at each of these vertices.

At 0 (0, 0)	Z = 3(0) + 4(0) = 0
At A (0, 24)	Z = 3(0) + 4(24) = 96
At B (10, 20)	Z = 3(10) + 4(20) = 110*
At C (20, 0)	Z = 3(20) + 4(0) = 60

Thus B (10, 20) gives the maximum value of Z. Hence the optimum solution is as follows :

$x_1=10$, $x_2=20$, $Z_{max} = 110$

11. Anil Ltd. manufactures models A, B and C which have profit contributions per unit of Rs 15, Rs 40 and Rs 60, respectively. The weekly minimum production requirements are 25 units for model A, 130 units for model B and 55 units for model C. Each type of recorder requires a certain amount of time for the manufacturing of component parts, for assembling and for packing. Specifically, a dozen units ,of model A require 4 hours for manufacturing, 3 hours for assembling and 1 hour for packaging. The corresponding figures for a dozen units of model B are 2.5, 4 and 2 and for a dozen units of model C are 6, 9 and 4.- During the forthcoming week, the company has available 130 hours of manufacturing, 170 hours of-assembling and 52 hours of packaging time. Formulate this problem as an LP model so as to maximize total profit to the company.

LP model formulation The data of the problem is summarized as follows:

Resources/Constraints	Models			Total Availability (hrs)
	A	B	C	
Production requirement (units)	25	130	A	
Manufacturing time (per dozen)	4	2.5	6	130
Assembling time (per dozen)	3	4	9	170
Packaging time- (per dozen)	1	2	4	52
Contribution per unit (Rs)	15	40	60	

Decision variables Let x1, x_2 and x3 = units of model A, B and C to be produced per week, respectively.

LP model

Maximize (total profit) = $15x_1 + 40x_2 + 60x_3$ subject to the constraints

(i) Minimum production requirement constraints

$$x_1 \geq 25$$
$$x_2 \geq 130$$
$$x_3 \geq 55$$

(ii) Manufacturing time constraint

$$(3x_1/12) + (4x_2/12) + (9x_3/12) \leq 130$$

(iii) Assembling time constraint

$$(x_1/12) + (2x_2/12) + (4x_3/12) \leq 170$$

(iv) Packaging time constraint

$$(x_1/12) + (2x_2/12) + (4x_3/12) \leq 52$$

and $x_1, x_2, x_3 \geq 0$

12. Consider the following problem faced by a production planner of Balkrishna Ltd. in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each may also be used for both types of bottles with some loss of efficiency. The manufacturing data is as follows:

Machine	8-ounce Bottles	16-ounce Bottles
A	100/minute	40/minute
B	60/minute	75/minute

The machines may be run for 8 hours per day, 5 days per week. Profit on an 8-ounce bottle is 15 paise and on a 16-ounce bottle 25 paise. Weekly production of the drink maynot exceed 3,00,000 bottles and the market may absorb 25,000, 8-ounce bottles and 7,000, 16-ounce bottles per week. The planner wishes to maximize his profit, subject of course, to all the production and marketing restrictions. Formulate this problem as an LP model to maximize total profit.

LP model formulation The data of the problem is summarized in the following table:

Constraints	Production		Availability
	8-ounce Bottle	16- ounce Bottle	
Machine A time	100/minute	40/minute	$8 \times 5 \times 60 = 2{,}400$ minutes
Machine B time	60/minute	75/minute	$8 \times 5 \times 60 = 2{,}400$ minutes

Production	1	1	3,00,000 units/week
Marketing	1	—	25,000 units/week
	—	1	7,000 units/week
Profit/unit (Rs)	0.15	0.25	-

Decision variables Let x1 and x_2 = units of 8-ounce and 16-ounce bottles, respectively to be produced weekly.

The LP model

Maximize (total profit) Z= 0.15x1 + 0.25x_2 subject to the constraints

(i) Machine time constraints

$$(x1/100) + (x2/40) = 2{,}400 \text{ and } (x1/60) + (x2/75) = 2{,}400$$

(ii) Production constraint

$$x1 + x_2 < 3{,}00{,}000$$

(iii) Marketing constraints

$$x1 < 25{,}000$$

$$x_2 < 7{,}000.$$

$$\text{and } x1, x2 > 0$$

13.: Parul Ltd. has two machines A and B. He manufactures two products P and Q on these machines. For manufacturing product P he has to use machine A for 3 hours and machine B for 6 hours, and for manufacturing product Q he has to use machine A for 6 hours and machine B for 5 hours. On each unit of P he earns Rs. 4 and on each unit of Q he earns Rs. 5. How many units of P and Q should be manufactured to get maximum profit ? Each machine maynot be used for more than 2100 hours.

Ans. : Suppose he manufactures x units of P and y units of Q. The following are the constraints :

$$3x + 6y \leq 2100$$

$$6x + 5y \leq 2100$$

$$x \geq 0, y \geq 0$$

and we have to find x and y such that the profit Z = 4x + 5y is maximum. Changing inequalities into equations, we get

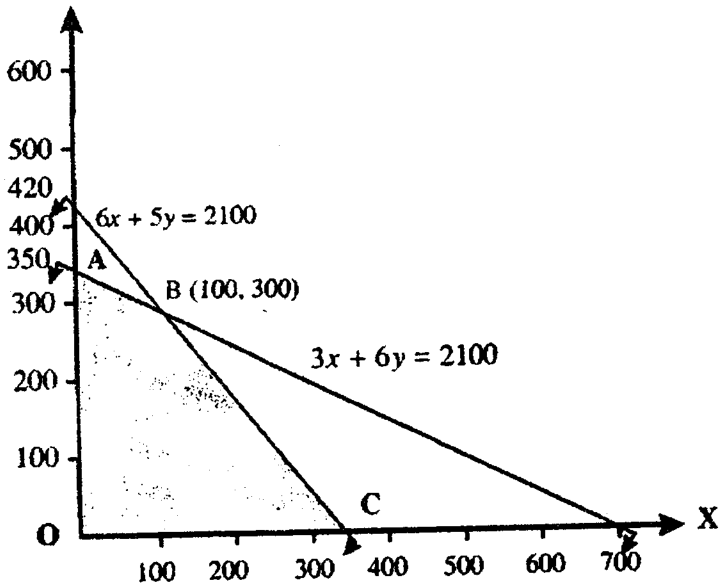
$3x + 6y = 2100$ $\qquad$ $6x + 5y = 2100$

$3x + 6y = 2100$

x	0	700
y	350	0

$6x + 5y = 2100$

X	0	350
y	420	0



From the given inequalities we get the convex polygon OABC. The vertices of this polygon give feasible solutions. We shall obtain the values of Z at each of these vertices

At O (0, 0) Z = 4 (0) + 5 (0) = 0

At A (0, 350) Z = 4 (0) + 5 (350) = 1750

At B (100, 300) Z = 4 (100) + 5 (300) = 1900*

At C (350, 0) Z = 4 (350) + 5 (0) = 1400

Thus at B (100, 300), Z is maximum.

The optimum feasible solution is

x = 100, y = 300, and Z_{max} = 1900.

Hence 100 units of P and 300 units of Q should be manufactured to get maximum profit.

14.: Amardeep Ltd. produces two types of machines A and B. There are two sections in his factory. In section I the assembling of parts is done and in section II the finishing of the product is done. The following are certain information available :

Section	No. of workers required	
	A	B
I	5	2
II	3	3

In section I not more than 180 workers may be employed and in section II not more than 135 workers may be employed. The number of B type machines are to be manufactured, double or less than that of A type machines. If each A type machine gives profit of Rs. 100 and B type machine gives profit of Rs. 150, find how many machines of each type the manufacturer should produce so as to obtain maximum profit.

Ans. : Suppose the manufacturer produces x machines of type A and y machines of type B. The given problem may be represented as follows :

Find x and y such that

$x, y \geq 0$

$5x + 2y \leq 180$

$3x + 3y \leq 135$

$y \leq 2x$

and the profit $Z = 100x + 150y$ is maximum, converting the inequalities into equations

$5x + 2y = 180$

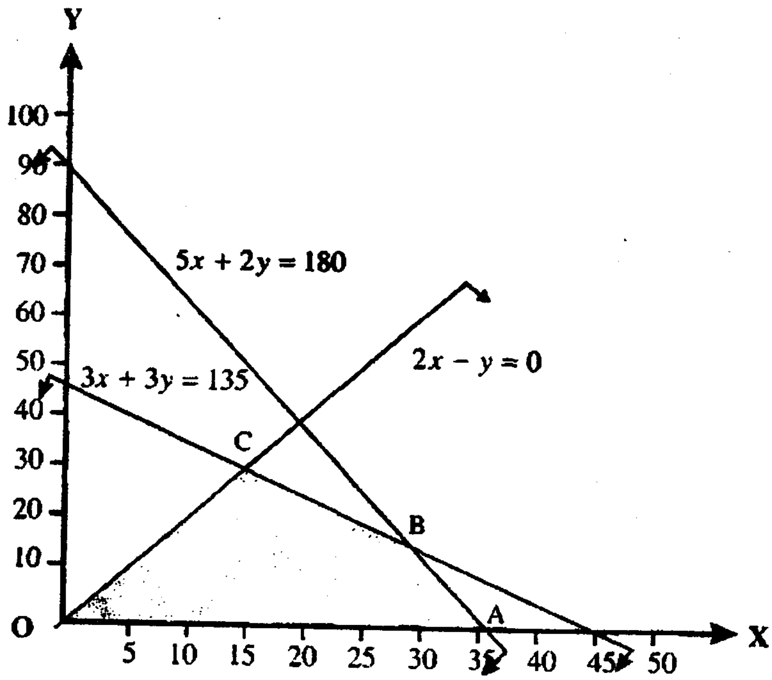
x	0	36
y	90	0

$3x + 3y = 135$

x	0	45
y	45	0

$y = 2x$

x	0	5
Y	0	10



By drawing the graphs of linear inequalities we get the region OABC. The feasible solutions are available at the vertices of this region we shall find the values of Z at each of these vertices

At 0 (0, 0) Z = 100 (0) + 150 (0) = 0

At A (36, 0) Z = 100 (36) + 150 (0) = 3600

At B (30, 15) Z = 100 (30) + 150 (15) = 5250

At C (15, 30) Z = 100 (15) + 150 (30) = 6000*

Thus at (15, 30) Z is maximum. Hence optimum solution is

$x = 15, y = 30, Z_{max} = 6000$

i.e. The manufacturer should produce 15 machines of type A and 30 machines of type B.

15. In Smita Ltd., a complete unit of a certain product consists of four units of component A and three units of component B. The two components (A and B) are manufactured from two different raw materials of which 100 .units and 200 units. respectively, are available. Three departments are engaged in the production process with each department using a different method for manufacturing the components per production run and the resulting units of each component are given below:

Department	Input per Run (units)		Output per Run (units)	
	Raw Material I	Raw Material II	Component A	Component B
1	7	5	6	4
2	4	8	5	8
3	2	7	7	3

Formulate this problem as an LP model to determine the number of production runs for each department which will maximize the total number of complete units of the final product.

LP model formulation Let $x1$, x_2 and x_3 = number of production runs for departments 1, 2 and 3, respectively.

Since each unit of the final product requires 4 units of component A and 3 units of component B, therefore maximum number of units of the final product maynot exceed the smaller value of

LP model formulation Let x_1, x_2 and x_3 = number of production runs for departments 1, 2 and 3, respectively.

Since each unit of the final product requires 4 units of component A and 3 units of component B, therefore maximum number of units of the final product cannot exceed the smaller value of

$$\left\{\frac{\text{Total number of units of A produced}}{4}; \frac{\text{Total number of units of B produced}}{3}\right\}$$

or

$$\left\{\frac{6x_1+5x_2+7x_3}{4} \text{ and } \frac{4x_1+8x_2+3x_3}{3}\right\}$$

Also if y is the number of component units of final product, then obviously, we have

$$\frac{6x_1+5x_2+7x_3}{4} \geq y \text{ and } \frac{4x_1+8x_2+3x_3}{3} \geq y$$

The LP model

$$\text{Maximize } Z = \text{Min}\left\{\frac{6x_1+5x_2+7x_3}{4}; \frac{4x_1+8x_2+3x_3}{3}\right\}$$

(i) Raw material constraints

$$7x_1 + 4x_2 + 2x_3 \leq 100 \quad \text{(Raw material I)}$$
$$5x_1 + 8x_2 + 7x_3 \leq 200 \quad \text{(Raw material II)}$$

(ii) Number of component units of final product constraints

$$6x_1 + 5x_2 + 7x_3 - 4y \geq 0$$
$$4x_1 + 8x_2 + 3x_3 - 4y \geq 0$$

and $x_1, x_2, x_3 \geq 0$

16. Ankur Ltd. wishes to plan an advertising campaign in three different media: "television, radio and a magazine. The purpose of the advertising is to reach as many potential customers as possible. Following are the results of a market study:

	Television			
	Prime Day (Rs)	Prime Time (Rs)	Radio (Rs)	Magazine (Rs)
Cost of an advertising unit	40,000	75,000	30,000	15,000
Number of potential customers reached per unit	4,00,000	9,00,000	5,00,000	2,00,000
Number of women customers reached per unit	3,00,000	4,00,000	2,00,000	1,00,000

The company does not want to spend more than Rs 8,00,000 on advertising. It is further required that

(i) at least 2 million exposures take place among women,

(ii) advertising on television be limited to Rs 5,00,000,

(iii) at least 3 advertising units be bought on prime day and two units during prime time; and

(iv) the number of advertising units on radio and magazine should each be between 5 and 10.

Formulate this problem as an LP model to maximize potential customer reach.

LP model formulation Let x_1, x_2, x_3 and x_4 = number of advertising units bought in prime day and time on television, radio and magazine, respectively.

The LP model

Maximize (total potential customer reach) $Z = 4{,}00{,}000x_1 + 9{,}00{,}000x_2 + 5{,}00{,}000x_3 + 2{,}00{,}000x_4$ subject to the constraints

(i) Advertising budget constraint

$$40{,}000x_1 + 75{,}000x_2 + 30{,}000x_3 + 15{,}000x_4 < 8{,}00{,}000$$

(ii) Number of women customers reached by the advertising campaign constraint

$3,00,000x1 + 4,00,000x2 + 2,00,000x3 + 1,00,000x4 > 20,00,000$

(iii) Television advertising constraints

$40,000x1 + 75,000x_2 < 5,00,000$

$x1 > 3$

$x_2 > 2$

(iv) Radio and magazine advertising constraints

$5 < x_3 < 10$

$5 < x_4 < 10$

and $x1, x_2, x_3, x_4 > 0$

17.: Abdul Ltd. has two iron mines. The production capacities of the mines are different. The iron ore may be classified into good, mediocre and bad varieties after certain process. The owner has decided to supply 12 or more tons of good iron, 8 or more tons of mediocre iron and 24 or more tons of bad iron per week.

The daily expense of first mine is Rs. 2000 and that of second mine is Rs. 1600. The daily production of each type of iron is given below :

Daily production			
Mine	good	mediocre	bad
1	6	2	4
2	2	2	12

To meet the supply most economically find the number of days for which the production in the mines should be carried out.

Ans. : Suppose the production in the first mine is carried out for x_1 days and that in the second mine is carried out for x_2 days. The problem may be represented as follows :

We want to find x_1, x_2 such that

$6x_1 + 2x_2 \geq 12,$

$2x_1 + 2x_2 \geq 8,$

$4x_1 + 12x_2 \geq 24$

$x_1, x_2 \geq 0$

$x_1 \leq 7, x_2 \leq 1$ (As number of days may not be more than 7)

and $Z = 2000x_1 + 1600x_2$ is minimum.

Converting inequalities into equations

$6x_1 + 2x_2 = 12$ $\quad\quad$ $2x_1 + 2x_2 = 8$

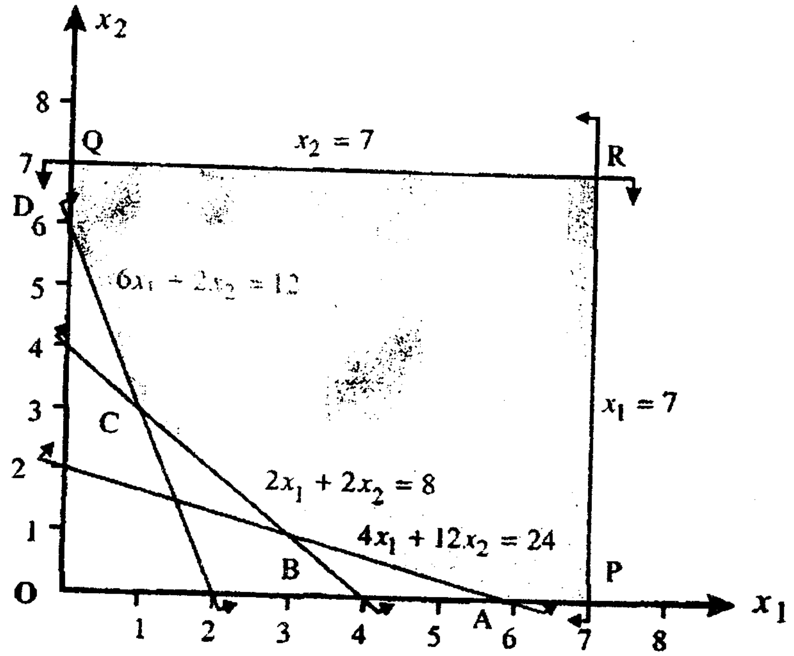
x_1	0	2

x_2	6	0

x_1	0	4
x_2	4	0

$4x_1 + 12x_2 = 24$

x_1	0	6
x_2	2	0



The graph of linear inequalities are drawn and the convex region PABCDQR is obtained.

The vertices of this polygon give the feasible solutions. We shall find the values of Z at each of these vertices.

At P (7, 0) Z = 2000 (7) + 1600 (0) = 14000

At A (6, 0) Z = 2000 (6) + 1600 (0) = 12000

At B (3, 1) Z = 2000 (3) + 1600 (1) = 7600

At C (1, 3) Z = 2000 (1) + 1600 (3) = 6800*

At D (0, 6) Z = 2000 (0) + 1600 (6) = 9600

At Q (0, 7) Z = 2000 (0) + 1600 (7) = 11200

At R (7, 7) Z = 2000 (7) + 1600 (7) = 25200

Thus Z is minimum at C (1, 3)

The optimum solution is

$x_1 = 1, x_2 = 3, Z_{min} = 6800$

Hence the first mine should work for 1 day and the second mine should work for 3 days per week.

18 : Two types of hens are kept in a Ravi poultry farm. A type ©! hen costs Rs. 20 each and B type of hen costs Rs. 30 each. A type of hen lays 4 eggs per week and B type of hen lays 6 eggs per week. At the most 40 hens may be kept in the poultry. Not more than Rs. 1050 is to be spent on the hens. How many hens of each type should be purchased to get maximum eggs ?

Ans.: Suppose x hens of type A and y hens of type B are purchased. The problem may be represented as follows :

We have to find the values of x and y such that

$x \geq 0, y \geq 0,$

$20x + 30y \leq 1050,$

$x + y \leq 40$

and $Z = 4x + 6y$ is maximum.

We obtain the following equations from the given inequalities :

$20x + 30y = 1050$

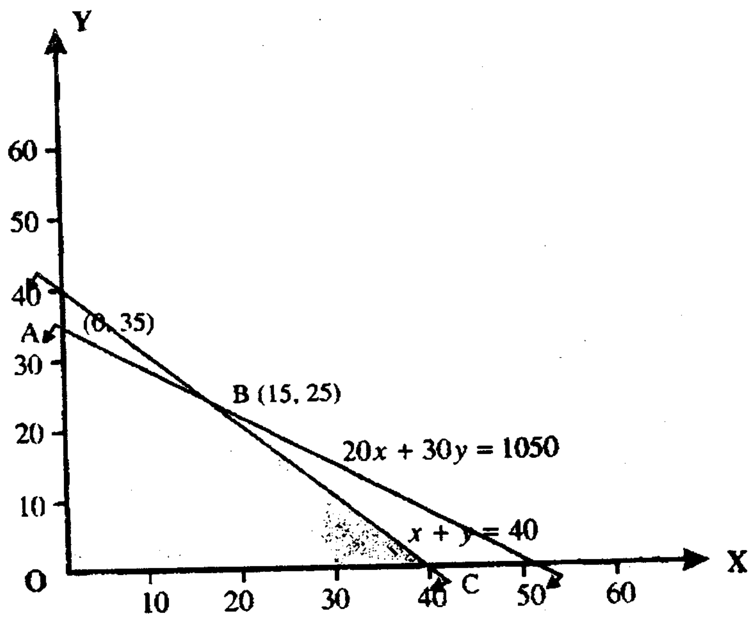
$x + y = 40$

$20x + 30y = 1050$

x	0	30
y	35	15

$x + y = 40$

x	0	40
y	40	0



The convex polygon OABC is obtained from the given inequalities. The vertices of this polygon give feasible solutions. We shall find the values of Z at each of these vertices :

At 0 (0, 0) $Z = 4(0) + 6(0) = 0$

At A (0, 35) $Z = 4(0) + 6(35) = 210$*

At B (15, 25) $Z = 4(15) + 6(25) = 210$*

At C (40, 0) Z = 4(40) + 6(0) = 160

The maximum value of Z is obtained at two points A and B, hence we get two solutions. Thus if he purchases 35 hens of type B he may get maximum 210 eggs or if he purchases 15 hens of type A and 2.5 hens of type B he may also get maximum 210 eggs.

19. Aniket Ltd. is planning to diversify its operations during the year "T998-99. The company has allocated capital expenditure budget equal to Rs 5.15 crore in the year 1998 and Rs 6.50 crore in the year 1999. The company has five investment projects under consideration. The estimated net returns at the present value and expected cash expenditures on each project in the two years are as follows:

Project	Estimated Net Returns (in '000 Rs}	Cash Expenditure (in '000 Rs)	
		Year 1998	Year 1999
A	240	120	320
B	390	550	594
C	80	118	202
D	150	250	340
E	182	324	474

Assume that the return from a particular project would be in direct proportion to the investment in it, so that, for example, if in a project, say A, 20% (of 120 in 1998 and of 320 in 1999) is invested, then the resulting net return in it would be 20% (of 240). This assumption also implies that individuality of the project should be ignored. Formulate this capital budgeting problem as an LP model to maximize the net return.

LP model formulation Let $x1$, x_2, x_3, x_4 and x_5 = proportion of investment in project A, B, C, D and E, respectively.

The LP model

Maximize (net return) = $240x1 + 390x_2 + 80x_3 + 150x_4 + 182x_5$ subject to the constraints

(i) Capital expenditure budget constraints

$120x1+ 550x_2+ 118x_3 + 250x4 +324x5 < 515$ [Foryear 1998]

$320x1+ 594x2+202x_3+340x4+474x5 <650$ [Foryear 1999]

(ii) 0-1 integer requirement constraints

$x1 < 1$; $x_2 < 1$ $x_3 < 1$; $x_4 < 1$; $x_5 < 1$ and $x1, x_2, x_3, x_4, x5 > 0$

20. Use the graphical method to solve the following LP problem for Gantakar Ltd.

Maximize $Z = 2x_1 + x_2$

subject to .the constraints

$x_1 + 2x_2 \leq 10$

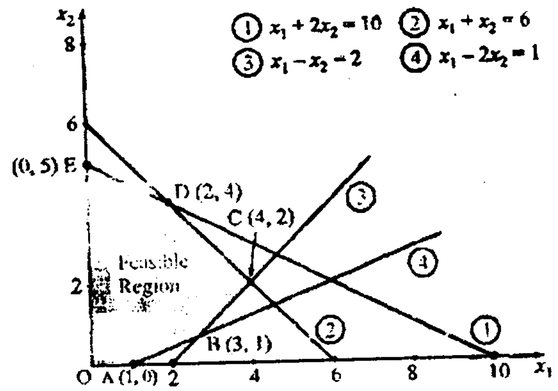
$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

and $x_1, x_2 \geq 0.$

Solution Plot on a graph each constraint by first treating it as a linear equation in the same way as discussed earlier. Use the inequality condition of each constraint to mark the feasible region as shown in Fig. 3.3. The feasible region is shown by the shaded area. Here it may be noted that we have not considered the area below the lines $x_1 - x_2 = 2$ and $x_1 - 2x_2 = 1$ for the negative values of x_2. This is because of the non-negativity condition, $x_2 > 0$, which implies that negative values of x_2 are not desirable.



Graphical Solution of LP Problem

The coordinates of extreme points of the feasible region are: O = (0, 0), A = (1, 0), B = (3, 1), C = (4, 2), D = (2, 4), and E = (0, 5). The value of objective function at each of these extreme points is as follows:

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 2x_1 + x_2$
O	(0, 0)	2(0) + 1(0) = 0
A	(1, 0)	2(1) + 1(0) = 2
B	(3, 1)	2(3) + 1(1) = 7
C	(4, 2)	2(4) + 1(2) = 10
D	(2, 4)	2(2) + 1(4) = 8
E	(0, 5)	2(0) + 1(5) = 5

The maximum value of the objective function Z = 10 occurs at the extreme point (4, 2). Hence, the optimal solution to the given LP problem is: $x_1 = 4$, $x_2 = 2$ and Max Z = 10.

21.-Use the graphical method to solve the following LP problem for Manibhadra Ltd.

Minimize $Z = 20x_1 + 10x_2$ subject to the constraints

$x_1 + 2x_2 < 40$

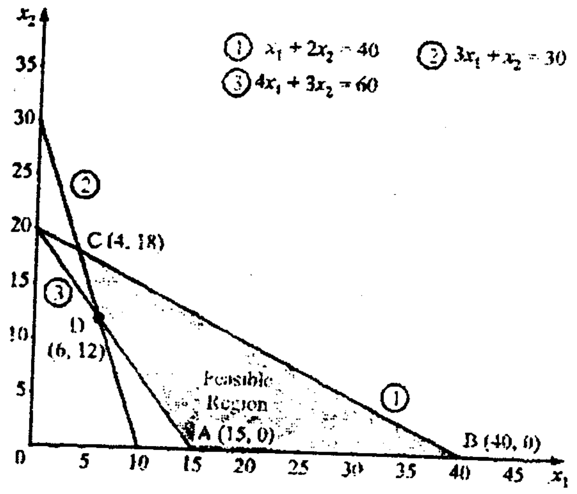
$3x_1 + x_2 > 30$

$4x_1 + 3x_2 \geq 60$

and $x_1, x_2 \geq 0$.

Solution Plot on a graph each constraint by first treating it as a linear equation. Then use the inequality condition of each constraint to mark the feasible region as shown below.

The coordinates of the extreme points of the feasible region are: A = (15,0), B = (40,0), C = (4,18) and D = (6, 12). The value of the objective function at each of these extreme points is as follows:



Graphical Solution of LP Problem

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 20\,x_1 + 10\,x_2$
A	(15, 0)	20(15) + 10(0) = 300
B	(40, 0)	20(40) + 10(0) = 800
C	(4, 18)	20(4) + 10(18) = 260
D	(6, 12)	20(6) + 10(12) = 240

The minimum value of the objective function Z = 240 occurs at the extreme point D (6, 12). Hence, the optimal solution to the given LP problem is: x, = 6, x_2 = 12 and Min Z = 240.

22. Use the graphical method to solve the following LP problem for Padmavati Ltd.

Maximize $Z = 2\,x_1 + 3x_2$, subject to the constraints

$x_1 + x_2 \leq 30$

$x_2 \geq 3$

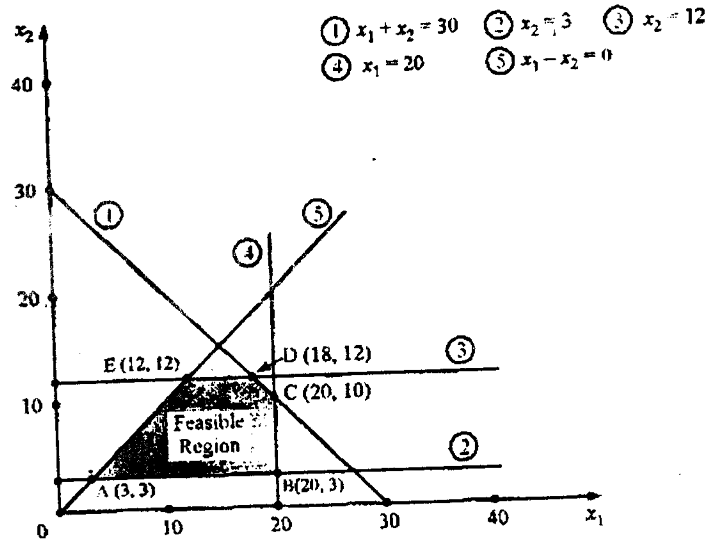
$0 < x_2 \leq 12$

$0 < x_1 < 20$

$x_1 - x_2 > 0$

and $x_1, x_2 > 0$.

Solution Plot on a graph each constraint by first treating it as a linear equation. Then use the inequality condition of each constraint to mark the feasible region as shown in.



Graphical Solution of LP Problem

The coordinates of the extreme points of the feasible region are: A = (3, 3), B = (20, 3), C = (20. 10), D = (18, 12) and E = (12, 12). The value of the objective function at each of these extreme points is as follows:

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 2x_1 + 3x_2$
A	(3, 3)	2(3) + 3(3) = 15
B	(20, 3)	2(20) + 3(3) = 49
C	(20, 10)	2(20) + 3(10) = 70
D	(18, 12)	2(18) + 3(12) = 72
E	(12, 12)	2(12) + 3(12) = 60

The maximum value of the objective function Z= 72 occurs at the extreme point D (18, 12). Hence, the optimal solution to the given LP problem is: $x_1 = 18$, $x_2 = 12$ and Max Z = 72.

23. Rupa Ltd. makes two products X and Y, and has a total production capacity of 9 tonnes per day,, X and Y requiring the same production capacity. The firm has a permanent contract to supply at least 2 tonnes of X and at least 3 tonnes of Y per day to another company. Each tonne of X requires 20 machine hours of production time and each tonne of Y requires 50 machine hours of production time. The daily maximum possible number of machine hours is 360. All the firm's output may be sold, and the profit made is Rs 80 per tonne of X and Rs 120 per tonne of Y. It is required to determine the production schedule for maximum profit and to calculate this profit.

Solution Let us define the following decision variables:

x_1 and x_2 = number of units (in tonnes) of product X and Y to be manufactured, respectively. Then the LP model of the given problem may be written as

Maximize (total profit) $Z = 80x_1 + 120x_2$ subject to the constraints

(i) Production capacity constraints

$x_1 + x_2 < 9$

$x_1 > 2; x_2 \geq 3$

(ii) Machine hours constraint

$20\,x_1 + 50x_2 < 360$

and $x_1, x_2 \geq 0$

For solving this LP problem graphically, let us graph each constraint by first treating it as a linear equation in the same way as discussed earlier. Then use the inequality condition of each constraint to mark the feasible region as shown in Fig. 3.10.

The coordinates of the extreme points of the feasible region are: A = (2, 3), B = (6, 3), C = (3, 6), and D = (2, 6.4). The value of the objective function at each of these extreme points is as follows:

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 80\,x_1 + 120x_2$
A	(2, 3)	80(2) + 120(3) = 520
B	(6, 3)	80(6) + 120(3) = 840
C	(3, 6)	80(3) + 120(6) = 960
D	(2, 6.4)	80(2) + 120(6.4) = 928

The maximum value of the objective function Z = 960 occurs at the extreme point C (3, 6). Hence the company should produce $x_1 = 3$ tonnes of product X and $x_2 = 6$ tonnes of product Y in order to yield a maximum profit of Rs 960.

24. Raviraj Ltd. produces two different models: X and Y, of the same product. Model X makes a contribution of Rs 50 per unit and model Y, Rs 30 per unit towards total profit.. Raw materials r_1, and r_2 are required for production. At least 18 kg of r, and 12 kg of r_2 must be used daily. Also at most 34 hours of labour are to be utilized. A quantity of 2 kg of r_1 is needed for model X and 1 kg of r_2 for model Y. For each of X and Y, 1 kg of r_2 is required. It takes 3 hours to manufacture model X and 2 hours to manufacture model Y. How many units of each model should be produced to maximize the profit?

Solution -Let us define the following decision variables:

x_1 and x_2 = number of units of model X and Y to be produced, respectively.

Then the LP model of the given problem may be written as:

Maximize (total profit) $Z = 50\,x_1 + 30x_2$ subject to the constraints

(i) Raw material constraints

$2\,x_1 + x_2 \geq 18$

$x_1 + x_2 \geq 12$

(ii) Labour hours constraint

$3x_1 + 2x_2 < 34$

and $x_1, x_2 \geq 0$

The problem solution is depicted graphically given below. The feasible region is shaded according to the inequality condition of each constraint.

The coordinates of extreme points of the feasible region are: A = (6, 6)', B = (2, 14), and C = (10, 2). The value of the objective function at each of these points is as follows:

Graphical Solution of LP Problem

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 50\,x_1 + 30\,x_2$
A	(6, 6)	50(6) + 30(6) = 480
B	(2, 14)	50(2) + 30(14) = 520
C	(10, 2)	50(10) + 30(2) = 560

25 Apurva Ltd. firm plans to purchase at least 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain at least 100 quintals of X-metal and not more than 35 quintals of Y-metal. The firm may purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weight in the scrap supplied by A and B is given below.

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is Rs 200 per quintal and that of B is Rs 400 per quintal. The firm wants to determine the quantities that it should buy from the two suppliers so that total cost is minimized.

Solution Let us consider the following decision variables:

x_1 and x_2 = quantity (in quintals) of scrap purchased from supplier A and B, respectively. Then the LP model of the given problem may be expressed as:

Minimize $Z = 200\,x_1 + 400x_2$

subject to the constraints

(i) Maximum purchase constraint

$x_1 + x_2 \geq 200$

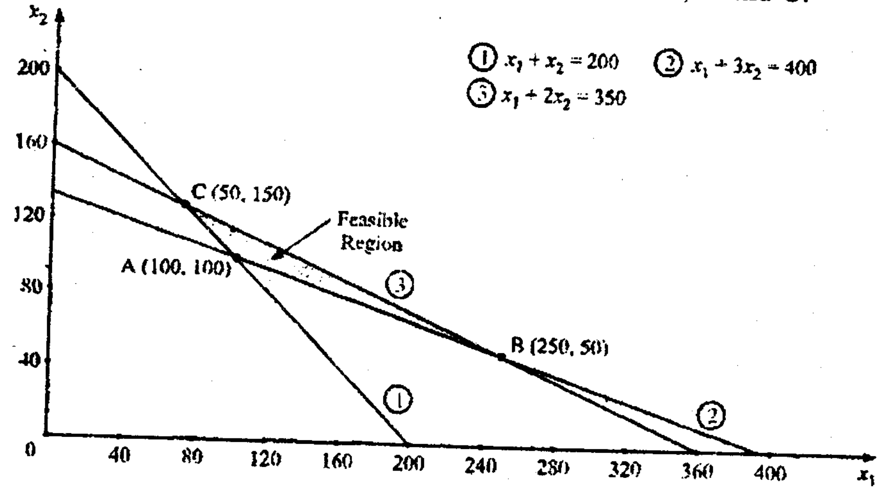
(ii) Scrap containing X and Y metal constraints

$x_{1/4} + 3\,x_2/4 \geq 10$ or $x_1 + 3\,x_2 \geq 400$

$x_{1/10} + x_2 < 35$ or $x_1 + 2\,x_2 < 350$

and $x_1, x_2 > 0$

The constraints are plotted on a graph as shown in below. The feasible region is shown by the shaded area and is .bounded by the corners, A, B and C.



Graphical Solution of LP Problem

The coordinates of the extreme points of the feasible region are: A = (100, 100), B = (250, 50), and C = (50, 150). The value of objective function at each of these extreme points is given below:

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 200x_1 + 400x_2$
A	(100. 100)	200(100) + 400(100) = 60,000
B	(250, 50)	200(250) + 400(50) = 70,000
C	(50, 150)	200(50) + 400(150) = 70,000

Since Z has the minimum value at the extreme point A (100, 100), the solution to the given problem is: $x_1 = 100$. $x_2 = 100$ and Min Z = Rs 60,000. That is, the firm should buy 100 quintals of scrap each from supplier A and B to minimize the total cost of purchase.

26 Use the graphical method to solve the following LP problem for Ravi Ltd.

Maximize $Z = 7x_1 + 3x_2$

subject to the constraints

$x_1 + 2x_2 \geq 3$

$x_1 + x_2 < 4$

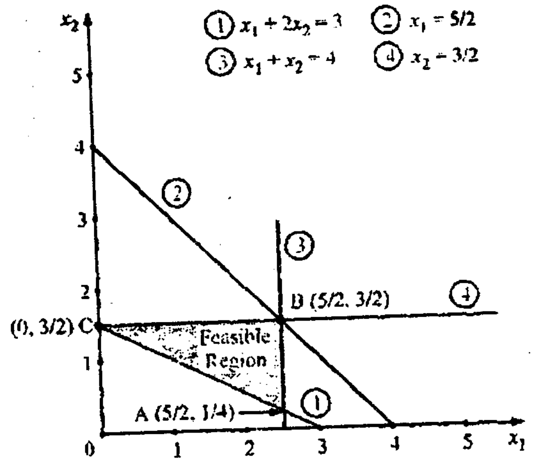
$0 < x_1 < 5/2$

$0 < x_2 < 3/2$

and $x_1, x_2 \geq 0$.

Solution Plot on a graph each constraint by first treating it as a linear equation. Then use the inequality condition of each constraint to mark the feasible region as shown in Fig. 3.7.

The coordinates of the extreme points of the feasible region are: A = (5/2,1/4), B = (5/2,3/2), and C = (0,3/2). The value of the objective function at each of these extreme points is as follows:



Graphical Solution of LP Problem

Extreme Point	Coordinates $(x_1\ x_2)$	Objective Function Value. $Z = 7x_1 + 3x_2$
A	(5/2. 1/4)	7(5/2) + 3(1/4) = 73/4
B	(5/2. 3/2)	7(5/2) + 3(3/2) = 22
C	(0, 3/2)	7(0) + 3(3/2) = 9/2

The maximum value of the objective function Z = 22 occurs at the extreme point B (5/2. 3/2). Hence, the optimal solution to the given LP problem is: $x_1 = 5/2$, $x_2 = 3/2$ and Max Z = 22.

27. Use the graphical method to solve the following LP problem for Kazi Ltd.

Minimize $Z = 3x_1 + 2x_2$

subject to the constraints

$5x_1 + x_2 \geq 10$

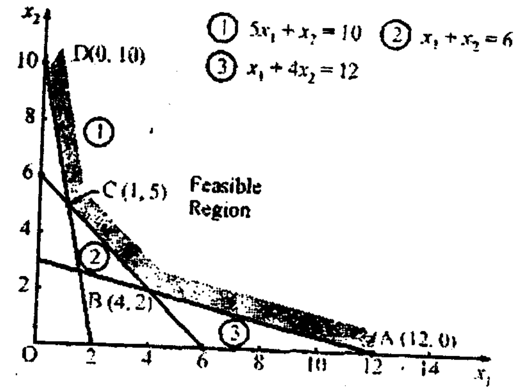
$x_1 + x_2 \geq 6$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 > 0.$

Solution Plot on a graph each constraint by first treating them as a linear equation in the same way as discussed earlier. Use the inequality condition of each constraint to mark the feasible region as shown below. It may once again be noted here that the area below the line $x_1 + 3x_2 = 10$ and x1 − x2 = 2 is not desirable, due to the reason that values of x1 and x2 are desired to be non-negative, i.e. x1 ≥ 0, x2 ≥ 0

The coordinates of the extreme points of the feasible region (bounded from below) are: A = (12, 0), B = (4, 2), C = (1, 5) and D = (0, 10). The value of objective function at each of these extreme points is as follows:



Graphical Solution of LP Problem

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 3x_1 + 2x_2$
A	(12, 0)	3(12) + 2(0) = 36
B	(4, 2)	3(4) + 2(2) = 16
C	(1, 5)	3(1) + 2(5) = 13
D	(0, 10)	3(0) + 2(10) = 20

The minimum value of the objective function Z = 13 occurs at the extreme point C (1, 5). Hence, the optimal solution to the given LP problem is: $x_1 = 1$, $x_2 = 5$, and Min Z = 13.

28. Use the graphical method to solve the following LP problem for Saraswati Ltd.

Minimize $Z = - x_1 + 2x_2$

subject to the constraints

$- x_1 + 3x_2 \leq 10$

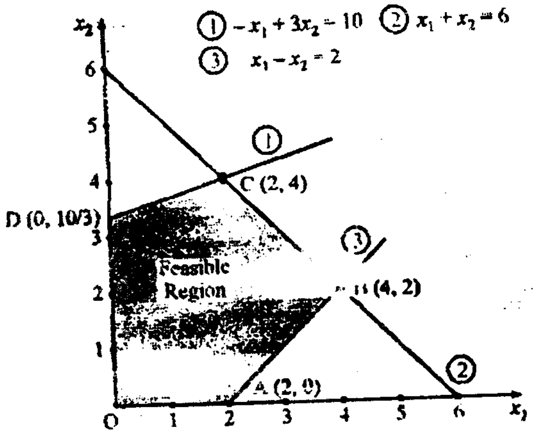
$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0.$

Solution Plot on a graph each constraint by first treating it as a linear equation as discussed earlier. Use the inequality condition of each constraint to mark the feasible region as given below. It may once again be noted here that the area below the lines $x_1 + 3x_2 = 10$ and $x_1 - x_2 = 2$ is not desirable, due to the reason that values of x_1 and x_2 are desired to be non-negative, i.e. $x_1 \geq 0, x_2 \geq 0$.

The coordinates of extreme points of the feasible region are: O = (0,0), A = (2,0), B = (4,2), C = (2,4), and D = (0, 10/3). The value of the objective function at each of these extreme points is as follows:



Graphical Solution of LP Problem

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = -x_1 + 2x_2$
O	(0, 0)	$-1(0) + 2(0) = 0$
A	(2, 0)	$-1(2) + 2(0) = -2$
B	(4, 2)	$-1(4) + 2(2) = 0$
C	(2, 4)	$-1(2) + 2(4) = 6$
D	(0,10/3)	$-1(0) + 2(10/3) = 20/3$

The minimum value of the objective function Z = - 2 occurs at the extreme point A (2, 0). Hence, the optimal solution to the given LP problem is: $x_1 = 2$, $x_2 = 0$ and Min Z = -2.

29. Navneet Ltd. is an investment company. To aid in its investment decision, the company has developed the investment alternatives for a 10-year period, as given in the following table. The return on investment is expressed as an annual rate of return on the invested capital. The risk coefficient and growth potential are subjective estimates made by the portfolio manager of the company. The terms of investment is the average length of time period required to realize the return on investment as indicated.

Investment Alternative	Length of Investment	Annual Rate of Return (Year)	Risk Coefficient	Growth Potential Return (%)
A	4	3	1	0
B	7	12	5	18
C	8	9	4	10
D	6	20	8	32
E	10	15	6	20
F	3	6	3	7

Cash	0	0	0	0

The objective of the company is to maximize the return on its investments. The guidelines for selecting the portfolio are:

(i) The average length of the investment for the portfolio should not exceed 7 years.

(ii) The average risk for the portfolio should not exceed 5.

(iii) The average growth potential for the portfolio should be at least 10%.

(iv) At least 10% of all available funds must be retained in the form of cash at all times.

Formulate this problem as an LP model to maximize total return.

LP model formulation Let xj = proportion of funds to be invested in the j'th investment alternative j =1,2,...,?)

The LP model

Maximize (total return) $Z= 0.03x1 + 0.12x_2 + 0.09x3 + 0.20x_4 + 0.15x5 + 0.06x_6 + 0.00x_7$ subject to the constraints

(i) Length of investment constraint

$$4x1 + 7x_2 + 8x_3 + 6x4 + 10x_5 + 3x_6 + 0x_7 < 7$$

(ii) Risk level constraint

$$x1 + 5x_2 + 4x_3 + 8x_4 + 6x5 + 3x_6 + 0x_7 < 5$$

(iiii) Growth potential constraint

$$0x1 + 0.18x_2 + 0.10x_3 + 0.32x_4 + 0.20x_5 + 0.07x_6 + 0x_7 > 0.10$$

(iv) Cash requirement constraint

$$x_7>0.10$$

(v) Proportion of funds constraint

$$xl + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1$$

and $x1, x_2, x_3, x_4, x_s, x_6, x_7 > 0$

30. Use graphical method to solve the following LP problem for Ram Ltd.: Maximize $Z = 3x_1 + 2x_2$

subject to the constraints

$x_1 - x_2 \geq 1$

$x_1 + x_2 > 3$

and , $x_1, x_2 \geq 0$.

Solution The constraints are plotted on graph as usual as shown in Fig. 3.18. The solution space is shaded and is bounded from below.

It is noted here that the shaded convex region (solution space) is unbounded from above. The two corners of the region are, A = (0, 3) and B = (2, 1). The value of the objective function at these corners is: Z(A) = 6 and Z(B) = 8.

Since the given LP problem is of maximization, there exist a number of points in the shaded region for which the value of the objective "function is more than 8. For example, the point (2, 3) lies in the region and the function value at this point is 12 which is more than 8. Thus, both the variables x_1 and x_2 may be made arbitrarily large and the value of Z also increases. Hence, the problem has an unbounded solution.

31. Use graphical method to solve the following LP-problem for Ratan Ltd.:

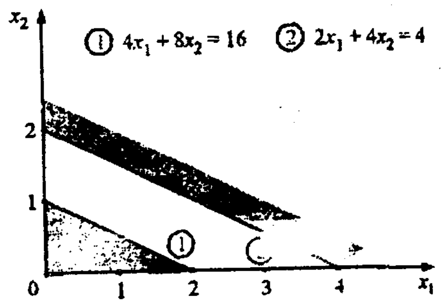
Maximize $Z = 6x_1 - 4x_2$ subject to the constraints

$2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

and $x_1, x_2 \geq 0$

Solution The constraints are plotted on graph as usual as shown below. There is no unique feasible solution space to get unique set of values of variables x_1 and x_2 that satisfies all the constraints. Hence, there is no feasible solution to this problem, because of conflicting constraints.



32: Maximize $Z = 2x_1 + x_2$ under the following constraints for Rafiq Ltd.

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2, \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1 - 2x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

Ans. : Converting inequalities into equations,

$x_1 + 2x_2 = 10$

x_1	0	10
x_2	5	0

$x_1 + x_2 = 6$

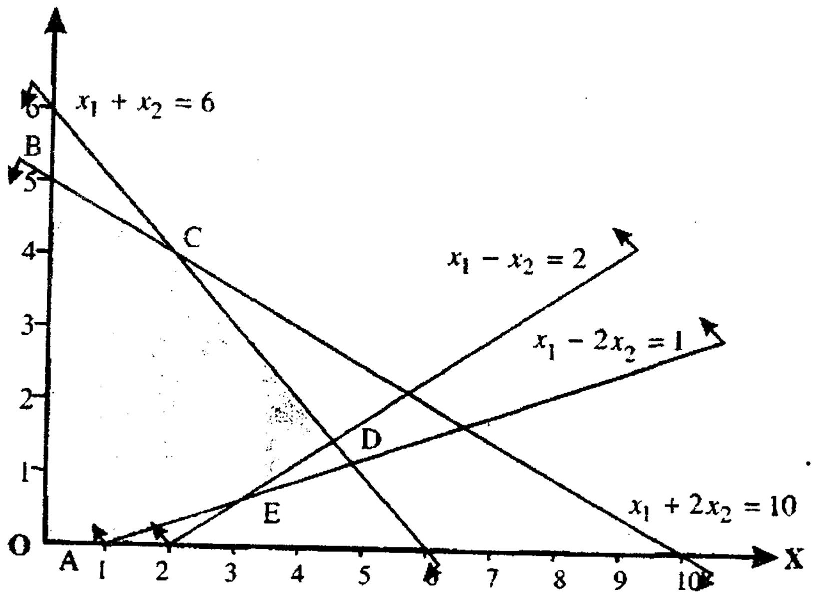
x_1	0	6
x_2	6	0

$x_1 - 2x_2 = 2$

x_1	3	2
x_2	1	0

$x_1 - 2x_2 = 1$

x_1	1	3
x_2	0	1



The convex polygon AOBCDE is obtained from the graphs of inequalities. The vertices of it give feasible solutions.

A (1, 0), O (0, 0), B (0, 5), C (2, 4), D (4, 2), E (3, 1)

We shall find out the values of Z at each vertex. For A (1, 0), Z = 2(1) + 0 = 2

For O (0, 0), Z = 2(0) + 0 = 0

For B (0, 5), Z = 2(0) + 5 = 5

For C (2, 4), Z = 2(2) + 4 = 8

For D (4, 2), Z = 2(4) + 2 = 10*

For E (3, 1), Z = 2(3) + 1=7

Z is maximum at point D. Therefore the optimal feasible solution is

$x_1 = 4$; $x_2 = 2$; $Z_{max} = 10$.

33. Use the graphical method to solve the following LP problem in following equation for Syam Ltd.

Maximize $Z = 15x_1 + 10x_2$

subject to the constraints

$4x_1 + 6x_2 \leq 360$

$3x_1 + 0x_2 \leq 180$

$0x_1 + 5x_2 \leq 200$

and $x_1, x_2 \geq 0$.

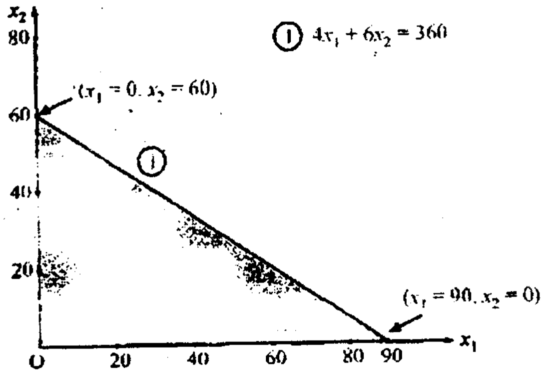
Solution. The given LP problem is already in mathematical form.

We shall treat x_1 as the horizontal axis and x_2 as the vertical axis. Each constraint will be plotted on the graph by treating it as a linear equation and then appropriate inequality conditions will be used to mark the .area of feasible solutions.

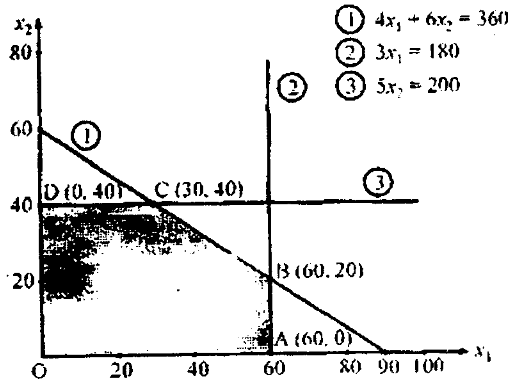
Consider the first constraint $4x_1 + 6x_2 \leq 360$. Treat it as the equation, $4x_1 + 6x_2 = 360$. The easiest way to plot this line is to find any two points that satisfy the equation, then drawing a straight line through them. The two points are generally the points at which the line intersects the x_1 and x_2 axes. For example, when $x_1 = 0$ we get $6x_2 = 360$ or $x_2 = 60$. Similarly when $x_2 = 0$, $4x_1 = 360$, $x_1 = 90$.

These two points are then connected by a straight line as shown in Fig. 3.1. But the question is: where are the points satisfying $4x_1 + 6x_2 \leq 360$. Any point above the constraint line violates the inequality condition. But any point below the line does not violate the constraint. Thus, the inequality and non-negativity condition may only be satisfied by the shaded area (feasible region) as shown below.

Similarly, the constraints $3x_1 \leq 180$ and $5x_2 \leq 200$ are also plotted on the graph and are indicated by the shaded area as shown below.



Graphical Solution of LP Problem



Graphical Solution of LP Problem

Since all constraints have been graphed, the area which is bounded by all the constraints lines including all the boundary points is called the feasible region (or solution space). The feasible region is shown by the shaded area OABCD.

Since the optimal value of the objective function occurs at one of the extreme points of the feasible region, it is necessary to determine their coordinates. The coordinates of extreme points of the feasible region are: O = (0, 0), A = (60, 0), B = (60, 20), C = (30, 40), D = (0, 40).

(ii) Evaluate objective function value at each extreme point of the feasible region as shown below:

Extreme Point	Coordinates (x_1, x_2)	Objective Function Value $Z = 15x_1 + 10x_2$
0	(0, 0)	15(0) + 10(0) = 0
A	(60, 0)	15(60) + 10(0) = 900
B	(60, 20)	15(60) + 10(20) = 1,100
C	(30, 40)	15(30) + 10(40) = 850
D	(0. 40)	15(0) + 10(40) = 400

(iii) Since we desire Z to be maximum, from 3(ii), we conclude that maximum value of Z = 1,100 is achieved at the point B (60, 20). Hence the optimal solution to the given LP problem is: $x_1 = 60$, $x_2 = 20$ and Max Z= 1,100.

Note: To determine which side of a constraint equation is in the feasible region, examine whether the origin (0, 0) satisfies the constraints. If it does, then all points on and below the constraint equation towards the origin are feasible points. If it does not, then all points on and above the constraint equation away from the origin are feasible points.

2.13 Practical Exercise

1. Find x and y such that $Z = 2x + 5y$ is maximum under the following constraints :

   $0 \leq x \leq 400$

   $0 \leq y \leq 500$

   $x + y \leq 600$

2. Maximize $Z = 20x_1 + 10x_2$ under the following constraints :

   $x_1, x_2 \geq 0$

   $30x_1 + x_2 \geq 30$

   $x_1 + 2x_2 \leq 540$

   $4x_1 + 3x_2 \geq 60$

3. Minimize $Z = 5x + 7y$ subject to

   $x, y \geq 0$

   $x + y \leq 4$

   $3x + 8y \leq 34$

   $5x + 2y \geq 10$

4. Minimize $Z = 5x + 3y$ under the following constraints :

   $0 \leq x \leq 3, \quad 0 \leq y \leq 3$

   $x + y \leq 6, 2x + 3y \geq 8$

5. Amrish Ltd. sells two different products A and B, making a profit of Rs 40 and Rs 30 per unit on them, respectively. They are produced in a common production process and are sold in two different markets. The production process has a total capacity of 30,000 man-hours. It takes three hours to produce a unit of A and one hour to produce a unit of B. The market has been surveyed and company officials feel that the maximum number

of units of A that may be sold is 9,500 units and that of B is 12,500 units. Subject to these limitations, products may be sold in any combination. Formulate this problem as an LP model to maximize profit.

6. The manager of Rassel Ltd. must decide on the optimal mix of two possible blending processes of which the input and output per production run are given as follows:

Process (units)	Input (units) Crude A	Crude B	Output Gasoline X	Gasoline Y
1	5	3	6	8
2	5	5	5	4

The maximum amount available of crude A and B are 200 units and 150 units, respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are Rs 350 and Rs 450, respectively. Formulate this problem as an LP model to maximize profit.

6. Ashish Ltd. places an order for a particular product at the beginning of each month and the product is received at the end of the month. The firm sells during the month from the stocks and it may sell any quantity.

The prices at which the firm buys and sells vary every month. The following table shows the projected buying and selling prices for the next four months:

Month	Selling Price (Rs) (During the Month)	Purchase Price (Rs) (Beginning of the Month)
April	—	80
May	95	75
June	62	61
July	75	—

The firm has no stocks on hand as on April 1, and does not wish to have any stocks at the end of July. The firm has a warehouse of limited size, which may hold a maximum of 150 units of the product.

Formulate this problem as an LP model to determine the number of units to buy and sell each month so as to maximize the profits from its operations.

7. In Parswanath Ltd. two machines A and B are used in manufacturing footballs and volleyballs. The machine A is to be used for 5 minutes and the machine B is to be used for 3 minutes to manufacture a foot-ball, while the machine A is to be used for 4 minutes and the machine B is to be used for 2 minutes to manufacture a volley-ball. Each machine may be used for at the most 3 hours a day. Each foot-ball gives a profit of Rs. 4 and each volley-ball gives a profit of Rs. 5. How many foot-balls and volleyballs should be manufactured per day to earn maximum profit ?

8. A furniture maker produces tables and chairs. In preparing a table 10 man hours are required and in preparing a chair 8 man hours are required. In a day maximum 300 man hours are available. One hour is required in polishing a chair or a table and 40 man hours are available for this. The profit on each table is Rs. 12 and that on each chair is Rs. 9. How many tables and chairs, he should produce to earn maximum profit ?

9. A milkman wants to purchase cows and buffaloes. He may accommodate at the most 20 animals in the available space. The daily expense on food and grass for a cow is Rs. 5 and that on a buffalo is Rs. 10. The milkman may spend at the most Rs. 136 a day. Each cow gives 5 litres milk and each buffalo gives 8 litres milk every day. How many cows and buffaloes should be purchased so as to get maximum quantity of milk ?

10. Amarprem Ltd. produces three models (I, II and III) of a certain product. He uses two types of raw materials (A and B) of which 4,500 and 6,000 units, respectively, are available. The raw material requirements per unit of the three models are as follows:

Raw Material	Requirement per Unit of Given Model		
	1	II	III
A	2	4	6
B	4	3	7

The labour time of each unit of model I is twice that of model II and three times that of model III. The entire labour force of the factory may produce equivalent of 2,500 units of model I. A market survey indicates that the minimum demand of the three models is: 500, 500 and 380 units, respectively. However, the ratios of the number of units produced must be equal to 3 : 2 : 5. Assume that the profit per unit of models I, II and III are Rs 60, 40 and 100, respectively. Formulate this problem as an LP model to determine the number of units of each product which will maximize profit.

11.Akar, furniture manufacturer makes two products: chairs and tables. Processing of these products is done on two machines A and B. A chair requires 3 hours on machine A and 6 hours on machine B. A table requires 6 hours on machine A and no time on machine B. There are 16 hours per day available on machine A and 40 hours on machine B. Profit gained by the manufacturer from a chair and a table is Rs 4 and Rs 10, respectively. What should be the daily production of each of the two products?

12. Adityavijay Ltd. produces two types of leather belts, say A and B. Belt A is of superior quality and B is inferior. Profit on the two are 40 and 35 paise per belt, respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all the belts were of type B, the company could produce 1,500 belts per day. But the supply of leather is sufficient only for 850 belts per day. Belt A requires a fancy buckle and only 450 of them are available per day. For belt B only 700 buckles are

available per day. How should the company manufacture the two types of belts in order to have a maximum overall profit?

13. Ratnakar Ltd. has a small plant which makes two types of automobile parts, say A and B. It buys castings that are machined, bored and polished. The capacity of machining is 26 per hour for A and 25 per hour for B, capacity of boring is 28 per hour for A and 35 per hour for B, and the capacity of polishing is 40 per hour for A and 25 per hour for B. Castings for part A cost Rs 3 and sell for Rs 5 each and those for part B cost Rs 3 and sell for Rs 6 each. The three machines have running costs of Rs 20, Rs 14 and Rs 17.50 per hour. Assuming that any combination of parts A and B may be sold. Formulate this problem as an LP model to determine the product mix which maximizes profit.

14. Anisha Ltd. produces two types of machines. For producing machine of type A, 2 tons of iron and 250 working hours are required and for producing machine of type B, 4 tons of iron and 160 working hours are required. The manufacturer has 950 tons of iron and 65,000 working hours at his disposal. If the profit on type A machine is rupees 550 and that on type B machine is rupees 800, find how many machines of type A and type B should be produced to get maximum profit.

15. Nilesh Ltd. manufactures two types of cupboards A and B. For manufacturing both the types of cupboards, two machines P and Q are to be used. The machine P is to be used for54 hours, and the machine Q is to be used for 2 hours to manufacture each cupboard of type A, and to manufacture each cupboard of type B the machine P is to be used for 4 hours and Q is to be used for 3 hours. The machine P may be used at the most for 24 hours and machine Q may be used at the most for 14 hours. A profit of Rs. 260 is earned on each cupboard of A type and that of Rs. 250 is earned on each cupboard of B type. How many cupboards of each type should be manufactured so as to earn maximum profit ?

16. On October 1, Kalajyot Ltd. received a contract to supply 6,000 units of a specialized product. The terms of contract require that 1,000 units be shipped in October; 3,500 units in November and 2,600 units in December. The company may manufacture 1,500 units per month on regular time and 750 units per month in overtime. The manufacturing cost per item produced during regular time is Rs 5 and the cost per item produced during overtime is Rs 5. The monthly storage cost is Re 1. Formulate this problem as an LP model so as to minimize total costs.

17. Reni Ltd. produces two types of leather belts; say A and B. Belt A is of a superior quality and belt B is of a lower quality. Profits on the two types of belts are Rs. 4 and Rs. 4 per belt respectively. Each belt of type A requires twice as much time as required by belt of type B. If all belts were of type B the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires a fency buckle and only 420 fency buckles are available per day. For belt of type B only 760 buckles are available per day. How should company manufacture two types of belts in order to have a maximum overall profit ?

18. Anish Ltd. produces two types of chairs A and B. There are two departments through which chairs are processed. The chair of type A takes 4 hours in the first department and 5.5 hours in the second department. Type B chair takes 4 hours in the first department and 6 hours in the second department. At the most 240 and 270 hours per week are respectively available in the two departments. At the most 40 chairs of type B may be sold per week. The profits on these chairs are Rs. 70 and Rs. 80 respectively. How many chairs of each type to be produced to get maximum profit.

19. Ravish Ltd. has two bottling plants, one located at 'G' and the other at T. Each plant produces three drinks: whisky, beer and brandy, named A, B and C, respectively. The number of bottles produced per day are as follows:

	Plant at	
Drink	G	J
Whisky	1,500	1,800
Beer	3,500	1,500
Brandy	2,500	5,500

A market survey indicates that during the month of July, there will be a demand of 20,000 bottles of whisky, 40,000 bottles of beer and 44,000 bottles of brandy. The operating cost per day for plants at G and J are 640 and 400 monetary units. For how many days should each plant be run in July so as to minimize the production cost, while still meeting the market demand? Solve graphically.

20. Consider a small plant which makes two types of automobile parts, ,say A and B. It buys castings that are machined, bored and polished. The capacity of machining is 25 per hour for A and 50 per hour for B, capacity of boring 'is 28 per hour for A and 45 per hour for B, and the capacity of polishing is 35 per hour for A and 26 per hour for B. Casting for part A costs Rs 5 each and for part B it costs Rs 3 each. They sell for Rs 5 and Rs 8, respectively. The three machines have running costs of Rs 20, Rs 15 and Rs 18.50 per hour. Assuming 'that any combination of parts A and B may be sold, what product mix maximizes profit?

21. A wine maker has a stock of three different wines with the following characteristics:

Wine	Proofs	Acids%	Specific Gravity	Stock (gallons)
A	28	0.32	1.80	20
B	35	0.20	1.08	34
C	33	0.30	1.05	22

A good dry table wine should be between 32 and 31 degree proof, it should contain at least 0.26% acid and should have a specific gravity of at least 1.06. The wine maker wishes to blend the three types of wine to produce as large a quantity as possible of a satisfactory dry table wine. However, his stock of wine A must be completely used in the blend because further storage would cause it to deteriorate. What quantities of wines B and C should be used in the blend. Formulate this problem as an LP model.

22. Sujan Ltd. machines and drills two castings X and Y. The time required to machine and drill one casting including machine set up time is as follows:

Casting	Machine Hours	Drilling Hours
X	4	3
Y	2	5

There are two lathes and three drilling machines. The working week is of 40 hours; there is no lost time and overtime. Variables costs for both castings are Rs 120 per unit while total fixed costs amount to Rs 1,200 per week. The selling price of casting X is Rs 300 per unit and that of Y is Rs 390 per unit. There are no limitations on the number of X and Y casting that may be sold. The company wishes to maximize its profit. You are required to (i) formulate a linear programming model for the problem and (ii) solve the problem graphically.

23. A diet for a sick person must contain atleast 4100 units of vitamins, 50 units of minerals and 1300 calories. Two foods A and B are available in the market at a cost of Rs. 50 and 35 respectively. One unit of A contains 250 units of vitamins. 1 unit of mineral and 40 calories and one unit of B contains 100 units of vitamins, 2 units of minerals and 40 calories. Solve the above problem graphically.

24. Akhil Ltd. has two machines A and B. He manufactures two products P and Q on these machines. For manufacturing one unit of product P, he has to use machine A for 4 hours and machine B for 10 hours and for manufacturing one unit of product Q he has to use machine A for 6 hours and machine B for 4 hours. On each unit of P he earns Rs. 14 and on each unit of Q he earns Rs. 20. How many units of P and Q should be manufactured to get maximum profit ? Each machine may not be used for more than 2500 hours.

25. Lal Ltd. wants to plan its advertising campaign for a new product. The company may spend at the most Rs. 65 lakhs for this. The new product may be useful to persons of middle age group. The company wants to use radio and prime-time television for advertisement purpose. The research department of the company has collected the following information regarding the cost and the effect of advertisement

Mode of advertisement	Cost per Unit (in rupees)	The number of middle age viewers influenced by an advertisement
Radic	50,000	5,00,000
Television	2,00,000	10,00,000

The company wants to reach to the maximum number of viewers of middle age group, and wishes to reach to at least 62 lakh persons through television. The company does not want to spend more than 15 lakhs on advertisement through radio. Using linear programming model, decide the number of times each media to be used for reaching to the maximum number of desired viewers.

36. Revati Ltd. assembles and markets two types of transistor radios, A and B. Presently 220 radios of each type are manufactured per week. You are advised to formulate the production schedule which will maximize the profits in the light of the following information:

Type	Total Component Cost per Radio (Rs)	Man-Hours of Assembly Time per Radio	Average Man-Minutes of Inspection and Correction Time per Radio	Selling Price per Radio (Rs)
A	200	16	11	400
B	160	6	30	280

The company employs 120 assemblers who are paid Rs 10 per hour actually worked and who will work up to a maximum of 50 hours per week. The inspectors, who are presently four, have agreed to a plan whereby they average 45 hours of work per week each. However, the four inspectors have certain other administrative duties which have been found to take up an average of 8 hours per week between them. The inspectors are each paid a fixed wage of Rs 900 per week.

Each radio of either type requires one speaker, the type being the same for each radio. However, the company may obtain a maximum supply of 650 in any one week. Their cost has been included in the components cost given for each radio in the table above. Only speakers actually used need be paid for. The only other cost incurred by the company are fixed overheads of Rs 23,000 per week.

37. Bhavik Ltd. owner wants to purchase two types of machines A and B. 2 workmen are required on machine A and 5 workmen are required on machine B. According to rules and regulations not more than 18 workmen may be engaged on each day and looking to the availability of space not more than 7 machines may be kept. Profit of Rs. 20 may be earned from machine A and that of Rs. 25 may be earned from machine B. How many machines of each type should be purchased so as to earn maximum profit ?

38. Albela Ltd. prepares two types of table lamps. For preparing first type of table lamp raw material of Rs. 5 is to be used and 3 hours are to be spared. For preparing the second type of table lamp raw material of Rs. 5 is to be used and 5 hours are to be spared. The manufacturer wishes to spend at the most Rs. 250 on raw material and he may spare at the most 185 working hours. A profit of Rs. 5 may be earned on each of first type of table lamp and that of Rs. 9 may be earned on each of the second type of table lamp. How many table lamps of each type should be prepared so as to earn maximum profit.

39. In a minibus at the most 16 passengers and maximum 48 kg. luggage may be taken. There are two types of passengers who travel in the bus :

(i) Those who have 4 kg. luggage and

(ii) Those who have 4 kg. luggage

The ticket for a passenger with luggage of 4 kg. is Rs. 3.50 and that for a passenger with luggage of 4 kg is Rs. 3.00. In what number of two types of passengers should be taken by the bus owner to get maximum income ?

40. Avinash Ltd. manufactures two types of bicycles A and B. For this two machines M and N are used. Machine M may be used for 70 hours and machine N may be used for 40 hours in a week. For manufacturing each bicycle of type A machine M is used for 1 hour and machine N is used for 3 hours, while for manufacturing each bicycle of type B machine M is used for 2.6 hours and machine N is used for 1 hour. The profit on each bicycle of type A is Rs. 90 and that on type B is Rs. 90. How many bicycles of each type should be manufactured per week so as to get maximum profit.

41. Prarthit has 80 plots in which he may build a house per plot. He wishes to build houses of two types A and B. He wishes that houses of B type should be at least three times than those of A type. If he desires the profit of Rs. 16,000 per house of type A and Rs. 10,000 per house of type B, to get maximum profit how many houses of each type should be built ?

42. A person requires three types of chemicals A, B and C for his garden. The minimum requirements of these chemicals are respectively 11, 12 and 12 units. These chemicals are available in liquid form in jars or in powder form in packets. A jar contains 2, 2 and 1 units of A, B and C respectively. A packet contains 1,2 and 4 units of A, B and C respectively. The price of a jar is Rs. 3 and that of a packet is Rs. 2. Find the number of jars and packets to be purchased so that the cost is minimum and the requirement is fulfilled.

43. Rupal Ltd. possesses two manufacturing plants, each of which may produce three products: X, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Following are the data on production per hour and costs together with current orders in hand for each product.

Plant	Products			Operating Cost
	X	y	Z	per Hour (Rs)
A	3	4	3	9
B	4	4	3	10
Orders on hand	50	24	60	

You' are required to use the graphic method to find out the number of production hours needed to fulfil the orders on hand at minimum cost.

44. Rashesh Ltd. manufactures two products: A and B. Product A yields a contribution of Rs 35 per unit and product B Rs 50 per unit towards fixed costs. It is estimated that sales of product A for the coming month will not exceed 20 units. Sales of product B have not been estimated but the company does have a contract to supply at least 10 units to a regular customer.

Machine hours available for the coming month are 130 and products A and B take 4 hours each, respectively, to produce. Labour hours available are 180 and products A and B take 4 hours and 6 hours of labour, respectively. Materials available are restricted to 40 units of two products while each uses one unit material per unit. The company wishes to maximize contribution. Using the graphic method, find the optimum product mix.

45. Lal Ltd. has been a producer of picture tubes for television sets and certain printed circuits for — radios. The company has just expanded into full-scale production and marketing of AM and AM-FM radios. It has built a new plant that may operate for 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM-FM radio will require 4 hours. Each AM radio will contribute Rs 40 to profits while an AM-FM radio will contribute Rs 90 to profits. The marketing department, after extensive research, has determined that a maximum of 16 AM radios and 10 AM-FM radios may be sold each week. Formulate this problem as an LP model to determine the optimal production mix of AM-FM radios that will maximize profits.

46. Mastram Ltd. wants to produce tv/o types of items P and Q. For manufacturing these items two machines for grinding and polishing are used. The grinding machine is to be used for 8 hours and the polishing machine is to be used for 4 hours for preparing each of P items, while the grinding machine is to be used for 5 hours and the polishing machine is to be used for 11 hours for preparing each of Q items. He has 4 grinding machines and 5 polishing machines available for the work. Each grinding machine and polishing machine may be used at the most for 60 hours a week. A profit of Rs. 33 may be earned on each of P items and a profit of Rs. 40 may be earned on each of Q items, How many items of P and Q should be manufactured every week so as to earn maximum profit ?

47. Atik Ltd. manufactures two types of motor cars Janta car and Deluxe car. According to Government restrictions the company has to manufacture at least 35 Janta cars and 20 Deluxe cars per week. Looking to the working capacity, the company maynot prepare more than 100 cars per week. For preparing one Janta car 20 machine hours and for preparing one Deluxe car 60 machine hours are required. The company may afford at the most 3800 machine hours per week. A profit of Rs. 1500 may be earned on each Janta car and that of Rs. 2500 may be earned on each Deluxe car. How many cars of each type should be manufactured so as earn maximum profit ?

48. Apurava furniture maker manufactures chairs and tables. He earns a profit of Rs. 30 on each chair and that of Rs. 50 on each table. Three machines M_1 M_2 and M_3 are to be used to prepare chairs and tables. The following table gives the information regarding availability of each machine and the hours required on each machine to prepare a chair and a table.

Machine	Chair	Table	Maximum available hours
M_1	4	3	36
M_2	5	2	50
M_3	2	6	60

How many chairs and tables should be manufactured to earn maximum profit ?

49. Suppose a media specialist has to decide on the allocation of advertising in three media vehicles. Let x_i be the number of messages carried in the media, $i = 1, 2, 3$. The unit costs of a message in the three media are Rs 1,700, Rs 750 and Rs 550. The total budget available is Rs 2,50,000 for the campaign period of a year. The first media is a monthly magazine and it is desired to advertise not more than one insertion in one issue. At least six messages should appear in the second media. The number of messages in the third media should strictly lie between 4 and 8. The expected effective audience for unit message in the media vehicles is shown below:

Vehicle	Expected Effective Audience
1	85,000
2	60,000
3	45,000

Formulate this problem as an LP model to determine the optimum allocation that would maximize total effective audience.

50. The owner of Sachin Sports Ltd. wishes to determine the number of advertisements to place in the selected three monthly magazines A, B and C. His objective is to advertise in such a way that total exposure to principal buyers of expensive sports goods is maximized. The percentage of readers for each magazine is known. Exposure to any particular magazine is the number of advertisements placed multiplied by the number of principal buyers. The following data may be used:

	Magazines		
	A	B	C
Readers	1 lakh	0.6 lakh	0.4 lakh
Principal buyers	15%	18%	7%
Cost per advertisement (Rs)	5,500	4,500	4,250

The budget amount is at the most Rs 2,50,000 for the advertisements. The owner has already decided that magazine A should have no more than six advertisements and that B and C each at least two advertisements. Formulate this problem as an LP model to determine the number of advertisements to be placed in each magazine.

51. In Shardul Ltd., the production consists of a machining process which takes raw materials and converts them into (unassembled) parts. These parts are then sent to one of the two divisions for assembly into the final product—Division 1 for product A and Division 4 for product B. Product A requires 50 units of raw material and 10 hours of machine processing time. Product B requires 80 units of raw material and 4 hours of machine processing time. During the period, 900 units of raw material and 80 hours of machine processing time are available. The capabilities of the two assembly divisions during the period are 6 and 10 units, respectively. The profit contribution per unit to profit and overhead (fixed costs) is Rs 260 for each unit of product 'A and Rs 120 for each unit of product B. With this information, formulate

this problem as a linear programming model and determine the optimal level of output for the two products using the graphic method.

52. Ratnmani is presenting a new book to be marketed. The book may be bound by either cloth or hard paper. Each cloth-bound book sold contributes Rs 25, and each paper-bound book contributes Rs 25. It takes 10 minutes to bind a cloth cover, and 9 minutes to bind a paperback. The total available time for binding is 80 hours. After considerable market survey, it is predicted that the cloth-cover sales will exceed at least 10,000 copies, but the paperback sales will be no more than 6,500 copies. Draw the feasible region which corresponds graphically to each constraint, and find the optimal solution.

53. Saroj Ltd. markets two feed mixes for cattle. The feed mix, Fertilex, requires at least twice as much wheat as barley. The second mix. Multiplex, requires at least twice as much barley as wheat. Wheat costs Rs 1.70 per kg, and only 1,000 kg are available this month. Barley costs Rs 1.27 per kg and 1,300 kg is available: Fertilex sells for Rs 1.80 per kg up to 99 kg and each additional kg over 99 sells for Rs 1.65. Multiplex sells at Rs 1.70 per kg up to 130 kg and each additional kg over 99 sells for Rs 1.55 per kg. Bharat Farms will buy any and all amounts of both mixes that PQR Feed Company will mix. Formulate this problem as a linear programming problem to determine the product mix that results in maximum profits.

54. Sutra Ltd. dealing with laminated sheets 'Gloss' in the western zone covering Maharashtra, Gujarat and Madhya Pradesh is considering to launch an advertisement campaign within a budget of Rs 2.75 lakh.- On. the basis of advertisement testing of the previous year, the company's research department has found that magazines and films are the ideal media for advertisements of laminated sheets. The company is not in a position to use the audiovisual medium due to limitation of funds. The magazines enjoying good recall in last year's campaign are Stardust, Filmfare, Reader's Digest and Madhuri. This is attributed to the effective -visual impact made by the good reproduction of the advertisements both in colour, and black and white.

55. A watch-dealer wishes to buy new watches and has two models M_1 and M_2. Model M_1 costs Rs. 250 and M_2 costs Rs. 390. His show case has space for 30 watches and he has Rs. 7,500 to spend. The watch dealer may make a profit of Rs. 20 in model M_L and Rs. 50 in model M_2. How many watches of each model should he buy to obtain maximum profit ?

56. Anup wants to start a printing press. Two types of machines A and B are available in the market. The machine of type A is available in Rs. 5,000 and that of type B is available in Rs. 12,000. The machine of type A prints 200 copies in an hour and that of type B prints 100 copies pet hour. 4 workers are required for machine A and only one worker is required for machine B. The person has Rs. 50,000 and he may employs at the most 16 workers. Represent the constraints of above problem as inequalities and also write the objective function.

57. The question paper of statistics is divided in two sections A and B. Each question in section A carries 4 marks and 4 minutes are required to solve it. Each question of section B carries 5 marks and 4 minutes are required to solve it. The total time given for the question paper is of 36 minutes and in all answers of 12 questions are to be given. Using graphical method find the number of questions to be attempted from each section to get maximum marks.

58. Upon completing the construction of his house, Mr Patel discovers that 150 square feet of plywood scrap and 89 square feet of white pine scrap are in un-usable form for the construction of tables and book-cases. It takes 16 square feet of plywood and 16 square feet of white pine to construct a book case. By selling the finished products to a local furniture store, Mr Patel may realise a profit of Rs 25 on each table and Rs 20 on each book-case. How may he most profitably use the left-over wood? Use graphical method to solve the problem.

59. Aniket Ltd. produces electric hand saws and electric drills, for which the demand exceeds his capacity. The production cost of a saw is Rs 6 and the production cost of a drill Rs 4. The shipping cost is 30 paise for a saw and 30 paise for a drill. A saw sells for Rs 9 and a drill sells for Rs 5.60: The budget allows a maximum of Rs 2,400 for production costs and Rs 130 for shipping costs. Determine the number of saw and drill that should be produced in order to maximize the excess of sales over production and shipping costs.

60. Rakesh Ltd. produces two types of products A and B. There are two sections in the factory. Each product of type A is to be processed for 4 hours in section I and 1 hour in section II. Each product of type B is to be processed for 1 hour in section I and 2 hours in section II. The section I may be used for maximum 104 hours per month and the section II may be used for maximum 75 hours per month. Each product of A gives a profit of Rs. 7 and each product of B gives a profit of Rs. 11. Find how many units of each type should be produced to earn maximum profit.

61. Abdul, a dealer of used scooters wishes to stock up his lot to maximize his profit. He may select scooters A, B and C which are valued on wholesale at Rs 5,500, Rs 7,000 and Rs 8,520 respectively. These may be sold at Rs 6,800, Rs 8,500 and Rs 10,500, respectively. For each type of scooter, the probabilities of sale are:

Type of scooter : A B C

Prob. of sale in 90 days : 0.9, 0.8, 0.6